PHOTOGRAMMETRIC MEASUREMENT OF FINAL PAY QUANTITIES IN HIGHWAY CONSTRUCTION

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by

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PURDUE UNIVERSITY
LAFAYETTE INDIANA

Final Report

TO: K. B. Woods, Director
Joint Highway Research Project

January 21, 1960

FROM: H. L. Michael, Assistant Director
Joint Highway Research Project

File: 1-4-15
Project: C-36-320

Attached is a final report entitled, "Photogrammetric Measurement of Final Pay Quantities in Highway Construction". This report has been prepared by Mr. Russell R. Johnson, graduate assistant on our staff under the direction of Professor R. D. Miles. The report was also utilized by Mr. Johnson as his thesis in partial fulfillment for the requirements of the M.S.C.E. degree.

This report summarizes the results of an attempt to apply photogrammetric methods to the determination of several final pay quantities in highway construction and to evaluate the accuracy obtained by comparison with quantities determined by normal field procedures. The results indicated some possibilities for the use of photogrammetric methods in the determination of final quantities.

A major portion of the cost of this project was paid through a Fellowship sponsored by the consulting engineering firm, Clyde E. Williams and Associates, Inc.

The report is submitted to the Board for the record.

Respectfully submitted,

Harold L. Michael

Harold L. Michael
Secretary

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Final Report

Photogrammetric Measurement of Final Pay Quantities in Highway Construction

by

Russell Richard Johnson
Graduate Assistant

Joint Highway Research Project
Project No. C-36-320
File No. 1-4-15

Purdue University
Lafayette, Indiana

January 21, 1960

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The author wishes to express his sincere appreciation for the assistance rendered him by Professor Robert D. Miles, Research Engineer, Joint Highway Research Project, in all phases of the study and in the preparation and review of the manuscript; to Professor Kenneth Curtis, School of Civil Engineering, for his advice on the control surveys; and to Dr. Irving W. Burr, Department of Mathematics, for his advice on the statistical aspects of the study.

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Finally, the author wishes to express his thanks to the personnel of the Joint Highway Research Project who assisted in the preparation of this report.

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ABSTRACT

Johnson, Russell Richard. MSCE., Purdue University, January 1960. Photogrammetric Measurement of Final Pay Quantities in Highway Construction. Major Professor: Robert D. Miles.

Photogrammetry is presently being used in the location and design of highways to achieve a saving in time, manpower, and cost. Considerable interest has been expressed in extending the use of photogrammetry to achieve a similar saving in the measurement of highway final pay quantities. The acceptance of photogrammetric methods for the determination of final pay quantities, however, is contingent upon their applicability in this respect and upon the accuracy that can be obtained for each type of measurement.

This thesis reports the results of an attempt to apply photogrammetric methods to the determination of several final pay quantities and to evaluate the accuracy obtained by comparison with quantities determined by normal field procedures. The final pay quantities concerned were earthwork, concrete pavement and appurtenances, paved side ditch, sodding, curbing, guard rail, and guide posts. The study was performed on a newly constructed section of highway approximately 10,000 feet in length. The photogrammetric plotting was accomplished with a standard six-inch focal length Kelsh

plotter at a scale of 1 inch = 50 feet.

Photogrammetric spot elevation readings for the final roadbed cross sections and field survey notes for the original terrain cross sections were used in combination to determine the earthwork quantities. Comparisons were analyzed on the basis of volumes for individual sections, for individual plotter models, for various classifications of excavation and embankment, for urban and rural classifications, and for the entire project. Although payment is made on the basis of excavation quantities only, embankment quantities were also considered as it was felt that they would aid in the evaluation of earthwork quantities in general. Computation of the earthwork quantities was accomplished with an IBM 650 electronic computer.

Since vertical accuracy is of great importance in the determination of earthwork quantities, a statistical analysis of vertical accuracy was undertaken by comparing photogrammetric centerline elevations with the corresponding elevations from the field survey.

Non-earthwork final pay items were delineated on the photogrammetric manuscript, and the quantities were determined by scaling or planimetering. These quantities were compared with the corresponding quantities determined by normal field procedures.

With limited regard for accuracy, it was found that the photogrammetric methods were applicable to the measurement of the aforementioned final pay quantities. The comparisons of these quantities, however, varied from good to bad. There was generally good agreement between the photogrammetric earthwork quantities and the corresponding quantities determined by field procedures, but poor agreement was noted in the comparisons of sodding and curbing. Adjustment of the photogrammetric earthwork quantities generally resulted in better agreement with field quantities.

Errors in photogrammetric elevation readings were normally distributed with a mean error not significantly different from zero. The calculated C-factor, a measure of vertical accuracy, was 1670, a comparatively high value.

Although the use of photogrammetric methods in the determination of final pay quantities still remained questionable, it was believed that the accuracy obtained in the earthwork quantity measurements would definitely warrant the use of photogrammetric methods in the location and design phases of highway construction.

PHOTOGRAMMETRIC MEASUREMENT OF FINAL PAY QUANTITIES IN HIGHWAY CONSTRUCTION

INTRODUCTION

Photogrammetry: Present Uses in Highway Engineering

Photogrammetry is defined as "the science or art of obtaining reliable measurements by means of photography" (1). Both ground and aerial photography may be used for this purpose, but aerial photography is employed almost exclusively in the application of photogrammetry to highway engineering.

The use of photogrammetric methods to supplement conventional highway location and design procedures has become a widespread practice during the past few years. The expanded highway program has induced highway departments to conscientiously apply photogrammetric methods to their engineering problems in order to keep within limitations of time, economy, and engineering manpower. Efforts in this area have been greatly reinforced by the recent development of relatively inexpensive photogrammetric plotting instruments and the integration of these instruments with automation and electronic computation. The experience and knowledge derived in this short period of time has proven photogrammetric methods to be satisfactory and in many respects superior to conventional methods.

Photogrammetric methods may be used in all of the stages of highway location and design. It is convenient to classify these stages as: (1) reconnaissance of alternate routes, (2) selection of the best route, and (3) design and estimation of quantities.

After the terminal points of a proposed highway have been selected, the task of location and design is begun. The first step is reconnaissance of the area between the terminal points to select possible alternate routes. In the past, ground reconnaissance on foot or visual observations from the air were relied upon as a means of selecting the best alternate routes, but experience has shown that the best alternates were not always chosen. Stereoscopic examination of small scale aerial photography over a large area enables the engineer to observe and record possibilities which might otherwise be overlooked. This regional view allows the consideration of many factors affecting location and insures a better selection of alternate routes. In the reconnaissance stage, rough measurements for comparative purposes may be obtained from small scale United States Geological Survey quadrangle sheets.

Comparison of the alternate routes and selection of the best is achieved by a detailed study of larger scale photography. Separate photographic flight line mosaics of each route at a scale of from 1 inch = 400 feet to 1 inch = 1000 feet are used. Tentative location lines and profiles are established and sample cross sections obtained by photogrammetric techniques as a basis for comparing the alternate routes.

Finally, the preliminary location line is fixed and design of the selected route accomplished by using very large scale photogrammetric contour maps or photogrammetrically obtained cross sections. It is in this stage that the full value of photogrammetric methods can be realized. With the aid of the contour map and stereoscopic examination of the photography, the engineer can pin-point the centerline of location; prepare profiles; establish grades; determine preliminary design earthwork quantities; study soil conditions, land use and drainage; determine sources of borrow materials and availability of suitable aggregates; and prepare fairly accurate quantity estimates. Except for a small amount of field surveying for basic control, the location and design from reconnaissance to preliminary estimation of quantities can be accomplished economically and with a minimum expenditure of time and manpower.

Through the various stages of highway location and design, photogrammetry is at first applied in a general manner and then becomes progressively more detailed, accurate, and specific to the final stages. In the reconnaissance stage, single photographs, stereo-pairs, and photo-mosaics are generally used while large scale photogrammetrically compiled contour maps or cross sections are used in the final stages.

Photogrammetric procedures are quite often employed for purposes other than those directly connected with location and design. They are well adapted to right-of-way studies since it is not necessary to disturb local property owners by the presence of ground survey crews or risk speculation on property values. Photogrammetric procedures have been useful aids in the operations and maintenance fields. They can be used to identify points of congestion and show relative use of roads in the vicinity of large traffic generators. Photography of the completed project reveals pavement condition, erosion, and drainage problems. It also affords an accurate record of the completed project for inventory surveys and for the settlement of legal problems.

The use of photogrammetric procedures to determine final pay quantities after completion of the project is of particular interest to this study. As far as is known, this method of determining final pay quantities has not been used to make final payment to the contractor; but, if accepted, it will make possible considerable savings in the cost, time, and manual effort involved in measuring and computing final pay quantities.

Final Payment Quantities

The present practice in Indiana regarding final earthwork quantities is to make payment on the basis of cubic yards of excavation as measured in the original position by taking cross sections before excavation is started and again after it is completed. Volumes are computed by the average end-area method. If the cost of excavation is specifically included in the payment for any item of work, the final cross sections are taken at the finished surface of the work. Payment for embankment is not ordinarily made on a unit volume basis, but is included in the various pay items of the contract (i.e., spreading and compacting of embankment material, labor and equipment).

Other final pay quantities such as pavement, curbing, paved side ditch, guard rail, and sodding are measured when complete in-place and accepted, with payment being made on a contract unit price per lineal foot or square yard (9).

The usual field procedure for obtaining cross sections consists of taking elevation readings to 0.1 foot on the terrain at right angles to the centerline. The horizontal distance of each reading from the centerline is usually measured to the nearest foot. Cross sections are taken at 100 foot intervals, or stations, and at significant breaks in the profile of the centerline. An engineer's level or a hand level may be used, depending upon the type of terrain and desired accuracy.

To estimate the area of a cut or fill section, the engineer plots the elevation readings from cross sectioning, both before and after construction, on coordinate paper and planimeters the area formed by connecting the elevation points by straight lines. The volume between two cross sections is determined by multiplying the averaged end-areas by the length between them. Electronic computers may be used to facilitate the computation of areas and volumes.

It is obvious that this procedure is at best an approximation of the true volume in that it represents only a sampling of the infinite number of sections and elevation points which would be necessary to represent the true geometry of a section of earthwork. The inaccuracies of this method are further emphasized by the fact that the average end-area formula for calculating volumes is not precisely correct; and, due to field practices, cross sections are rarely taken at exactly right angles to the centerline. Add to this the blunders that sometimes occur in any surveying procedure, and it becomes apparent that the present methods of estimating earthwork quantities are far from ideal.

Lineal measurements of guard rail, sodding, curbing, and other pay items are usually made with a metallic tape to the nearest foot or tenth of a foot. These measurements are not necessarily in a horizontal plane.

The proposed procedure for determining earthwork quantities photogrammetrically is a direct analogue of the field survey method. Cross section lines are drawn on the photogrammetric manuscript at right angles to the centerline. Spot elevations along the cross section lines are read directly from the photogrammetric plotter or the elevations may be interpolated from photogrammetrically established contours on the manuscript. For final pay measurements, it is generally agreed that spot elevations would be more desirable since they can be made with at least twice the precision of the interpolated readings (7). The coordinates of each reading are written on the manuscript at the position of the reading. Computation of volume quantities is made in the normal manner.

The ease and facility of reading elevations from the plotter, as opposed to the laborious and time consuming field methods, motivates the measurement of more cross sections and more elevations per cross section. This is highly desirable since, as indicated previously, the accuracy of the terrain representation is a function of the density of elevation readings. Elevation data collection may be further expedited by an electronic linkage mechanism which permits the horizontal and vertical coordinates of a point reading to be automatically punched into standard electronic computer punch cards. The plotter operator need only concentrate on placing the floating mark; the data is recorded by merely pushing a button.

In connection with the measurement of non-earthwork quantities, it may be said that the physical dimensions of any object identifiable to its full extent in the plotter model can be measured. The measurement of these quantities, however, is not always in the horizontal and vertical system of coordinates: a condition which calls for special allowances when making photogrammetric measurements since direct measurements from the plotter are always horizontal and vertical.

The cost, time, and manual effort involved in measuring final pay quantities would be considerably reduced if these quantities could be obtained by photogrammetric methods. The use of these methods, however, is contingent upon the accuracy which can be obtained for each type of measurement. Large sums of money are paid to the contractor on the basis of final pay measurements; it is mandatory that these measurements be accurate and reliable. In light of the savings and advantages of photogrammetric methods over conventional methods, a study to evaluate the photogrammetric accuracy of final pay quantities would seem appropriate.

Purpose

This study is an attempt to measure final pay quantities by photogrammetric methods and to evaluate the accuracy of such measurements. The quantities to be measured are earthwork, concrete pavement, concrete pavement appurtenances, paved side ditch, sodding, guard rail, guide posts and curbing. The accuracy obtained in measuring these quantities will be evaluated on the basis of comparisons with actual field measurements taken for final payment to the contractor.

PREVIOUS INVESTIGATIONS

The use of photogrammetric procedures to solve highway location problems and to determine design quantities is a well established practice in many state highway departments and private consulting firms, but the concept of determining final pay quantities by photogrammetric procedures is relatively new. Many authorities in the highway industry have proposed that the use of photogrammetric methods be extended to include the determination of final pay quantities for earthwork and, as a few have suggested, to final pay quantities other than earthwork quantities. Little work has been done to actually test the accuracy and reliability of photogrammetric methods in this application, however.

As early as 1957, Ohio reported flying three road projects for final measurements, and a later report outlined the procedure by which photogrammetric final pay measurements were to be made "if requested" (8,6). This procedure consists of obtaining large scale photography of the job as soon as the contractor has completed fine grading and seeding. All sections are measured photogrammetrically to obtain an accurate record of construction "as built" whether needed for payment or not. A duplicate of the original cross section overlay is prepared and the work limits and control elevations as indicated by the design sheets are added. The plotter operators measure spot elevations for the final cross sections on this overlay to at least the work limits and as much farther as earthwork movements indicate. The final cross sections are then plotted on copies of the cross sections taken before construction to check the tie-in edges of the sections. The extent of seeding, sodding, borrow and other pay items is plotted and turned over to the construction engineer for pay quantity calculations.

Unfortunately, the accuracy of this procedure is left open to conjecture since no comparisons of the photogrammetric measurements with normal methods of measurement were reported.

Early in 1958, the California Division of Highways reported a study on map accuracy which may be considered relevant to the problem of final pay measurements (4). This study was a statistical analysis of the vertical accuracy of photogrammetric mapping derived from a comparison of field elevations interpolated from contour maps. An analysis of the results of twelve projects indicated close agreement between the frequency distribution of errors and the normal probability distribution. The report concluded that present map specifications, which call for 90 percent of the readings tested to be within one-half contour interval, are inadequate and that the standard deviation, by itself, would be an equally inadequate measure of accuracy since systematic errors would not be revealed. From the standpoint of the highway engineer, it was felt that satisfactory specifications for mapping should embody both the arithmetic mean and the standard deviation.

The study also reported earthwork quantity comparisons for several projects based on contour map interpolation of elevation readings. The errors ranged from 0.1 percent to 5.4 percent in excavation and from .03 percent to 7.6 percent in embankment. It noted that this close agreement, even though the mapping was relatively poor in many cases, was due to the compensation of plus and minus errors.

Further elaboration on the accuracy of photogrammetric earthwork quantities was provided by two recent studies by the California Division of Highways. In the first study earthwork quantities from a 3000 foot experimental section were measured by two field surveys and six photogrammetric surveys (5). One of the field surveys was made with sufficient precision to serve as a control for measuring the accuracy of the other surveys. Three of the photogrammetric surveys were made by reading spot elevations along the cross section lines and the other three resulted from interpolating elevations along the cross section lines from a photogrammetrically prepared contour map of two-foot contour intervals. A 1 inch = 50 feet scale was used for all photogrammetric plotting.

A wide variation in accuracy was noted in measuring the elevation of centerline stations and slope stakes by the photogrammetric surveys. The errors in elevations between the control survey and the photogrammetric surveys ranged from -2.4 feet to 1.8 feet. The standard deviations of these surveys, calculated on the basis of deviations from the arithmetic mean error, ranged from .19 foot to .62 foot. The mean of the errors ranged from .03 foot to .32 foot. Earthwork quantities for this study as computed from the field control survey were 63,167 cubic yards of excavation and 29,152 cubic yards of embankment. Errors in quantities from the photogrammetric survey ranged from 0 to 2.5 percent in excavation and from 0.8 to 7.9 percent in embankment. An analysis of this variation showed that there was a close relationship between the errors in centerline elevations and the errors in earthwork quantities. Further investigation indicated that systematic errors were operative which varied from model to model and even within individual models, but tended to remain fairly constant for a small area, such as a single cross section. These relationships suggested the possibility of adjusting the earthwork quantities by raising or lowering the entire cross section at each station by an amount equal to the error at the centerline. After this adjustment was made, the errors in earthwork quantities of the photogrammetric surveys were from 0 to 0.8 percent in excavation and from 0.1 to 1.3 percent in embankment.

The conclusions of this study were that (1) "Adjusted quantities from all of the photogrammetric surveys were within limits generally considered tolerable for pay quantities," and (2) "the method of adjusting photogrammetric quantities by use of a centerline profile appears to have considerable potential value as a means of obtaining pay quantities with a minimum expenditure of manpower" (5).

The second study of three sections totaling 10.7 miles in length was made under actual field conditions and generally confirmed the results of the first study (3). The purpose of this study was to further test the theory that the accuracy of photogrammetric earthwork quantities could be greatly improved by centerline adjustments.

Again, a conventional field survey was used as a control for testing the accuracy of the photogrammetric survey and, as before, adjustments were made by raising or lowering all elevation readings at each cross section by an amount equal to the error in elevation from the field survey at centerline. Elevations along cross section lines were taken from photogrammetric maps at a scale of 1 inch = 50 feet with two-foot contour intervals.

For purposes of comparison, the three sections were divided into ten segments, each approximately one mile in length. The errors in excavation for the ten segments before adjustment ranged from 0.3 percent to 5.4 percent with an average of 2.5 percent. After adjustment these errors were reduced to a range of 0.0 percent to 1.8 percent with an average of 0.5 percent. The range for embankment quantities was from 0.9 to 9.7 percent with an average of 3.1 percent before adjustment and 0.1 to 1.8 percent with an average of 0.6 percent after adjustment. Comparisons were also made to show differences before and after adjustment for large individual excavations and embankments and for sections 1,000 feet in length.

A comparison of earthwork quantities using cross sections taken from photogrammetric mapping at a one foot contour interval and a scale of 1 inch = 50 feet with cross sections by field survey methods on six and one-half miles of the Northern Illinois Toll Highway was reported by Clyde E. Williams and Associates, Inc. (personal correspondence). Although this study was concerned with the estimation of unclassified excavation for bid purposes, it is an indication of the accuracy and reliability that might be expected in final pay quantity determination under similar field conditions.

Templates of the finished cross sections were plotted on the original terrain sections obtained from the photogrammetric contour map. Final photogrammetric cross sections were not taken after excavation was completed. Field cross sections were taken both before and after excavation.

The errors in the photogrammetric quantities for this project were larger than those reported by the California studies. This may be explained, in part, by the extremely flat terrain and attendant shallow cut sections and the final shaping of the back slopes after the contour map had been prepared.

The total excavation by field measurements was 144,827 cubic yards compared to 131,940 cubic yards by photogrammetric measurements for an 8.9 percent error. However, the comparison of total yardage is misleading since one section of shallow cut was in error 42 percent while another section of deeper cut was in error only three percent. It was felt that the latter section gave a more realistic picture for comparative purposes as this area was of a more controlled nature, regarding cut limits, than anywhere else on the job.

A survey of relevant literature failed to reveal any investigations on the accuracy of final payment quantities other than earthwork quantities and, as far as is known, this study is the first attempt to concentrate on the general problem of the accuracy involved in the photogrammetric measurement of several types of final pay quantities.

SCOPE

Study Location

Several factors were considered in the selection of a section of highway on which to perform the investigation. The study site had to be recently completed construction and of sufficient length to provide an adequate statistical base for accuracy evaluations. The topography, land use, and proximity to Lafayette were deemed desirable considerations, but not necessarily of a controlling nature.

The most important factor considered was that of finding a section of highway of recent construction. This is because weathering, traffic use, and vegetation tend to make photogrammetric identification and delineation of construction items difficult after the highway has been in use for any period of time. Erosion quickly disfigures side slopes and ditches, and traffic use soon obscures the sharp definition of pavement edges and side road entrances. Tree foliage and weeds may wholly or partially conceal terrain and ground objects, especially during the summer months, and make the task of photogrammetric plotting particularly difficult.

The length of the study section would, of course, depend upon the frequency of occurrence of the various pay items.

It was felt, however, that a study section of approximately two miles length would provide enough measurements of the various pay quantities to permit a sound evaluation of accuracy. This length would allow hundreds of comparisons of individual earthwork quantities and permit a statistical study of vertical accuracy and some of its ramifications as applied to earthwork quantities.

The study area was selected on the basis that it should also include variable topography and different land uses. These variables are likely to be encountered in a highway construction project of large size and may present quite different photogrammetric interpretation and measurement problems.

A newly constructed section of State Road 27 beginning at Liberty, Indiana and extending northward about five miles was selected as a suitable location for the study. The Indiana State Highway Department assisted in making the selection. The actual study was performed on a 9,643 foot length of this section beginning at station 445+00 on Westcott Street in Liberty and ending at station 538+55 north of town. A flight line mosaic of the study section is shown in Plate I.

The construction on State Road 27 was primarily of a relocation nature although some of the highway was rebuilt to higher standards in the same position as before construction. Construction was completed in the fall of 1958.

About 800 feet of the study section may be assigned an

FLIGHT LINE MOSAIC OF STUDY AREA

PLATE 1

urban land use classification as it is within the city limits of Liberty. The new highway is of two-lane construction with twelve foot lanes. The pavement and side road entrances are constructed of Portland cement concrete. Private drive entrances within the city limits are also paved with concrete. Large embankments have protective guard rail and hazardous locations are marked with guide posts. There are two relatively large drainage structures and several paved side ditches to accommodate runoff. Grades and curvatures are small, reflecting the high standards of design which generally prevail.

The terrain may be described as rolling although there are some flat reaches. This is typical of the heavily dissected Wisconsin-age till plain on which the study area is located. The elevation readings ranged from 927.2 feet to 1005.9 feet above sea level. Some of the excavations and embankments were relatively large as would be expected in terrain of this nature.

The ground cover at the time of flight exposure in December consisted mainly of crop stuble and pasture on the tillable soil and wooded areas near the streams. Residential buildings, lawns, and shade trees prevailed within the city limits. All vegetation and trees within the limits of construction had been removed, except for the sodding placed after construction and a few small areas near the outside limits of construction in which trees obscured the view.

Some snow cover was present in the side ditches and on the west slopes of excavations.

The study section generally complied with the conditions which were thought necessary or desirable in the selection of a good location for the investigation. Perhaps the only disagreeable aspect was the long travel time involved in visiting the site to collect control data.

Quantities Measured

It was decided early in the study that the quantities which could possibly be measured were earthwork, concrete pavement, concrete pavement appurtenances, paved side ditch, sodding, guard rail, guide post, and curbing. Only items which could be identified and delineated from air-photos were considered. Payment for such quantities had to be on the basis of dimension measure rather than weight measure. On this premise, a quantity measurement such as seeding would not qualify for photogrammetric measurement since payment for this item is on the basis of weight measurement of seed, fertilizer and straw mulching. Nor would base course materials lend themselves to photogrammetric measurement since these quantities are not visible from air-photos.

The most important quantity under investigation was earthwork. Although payment to the contractor in Indiana is made on the basis of cubic yards of excavation only, it was felt that the measurement of embankment quantities would serve as an indication of the accuracy and reliability of earthwork measurements in general. Two relatively large borrow pits adjacent to the highway at stations 508 and 530 were included under the classification of special borrow in the excavation measurements. Excavations and embankments for side road entrances and private drives were not measured since the original terrain data were not available in many instances. Grade "B" special borrow, which is granular material used primarily for special fill at structures, was not measured for the same reason. The amount of Grade "B" special borrow was very small, and it is doubtful if the borrow pits for this material could be identified and measured, even if they were within the limits of photo coverage.

Cross sectional areas for earthwork quantities were determined by a combination of photogrammetric and conventional methods. The original cross sections of the terrain before construction were procured by field survey methods; whereas, the template sections (i.e., cross sections of the finished road bed) were procured by photogrammetric spot elevation readings. Field level books from the highway department supplied the data for the original cross sections. Although it would have been possible, and perhaps desirable, to determine the initial terrain data by photogrammetric methods, the time interval from staking of the centerline to completion of construction would have precluded such a procedure for this study.

It was recognized that the plotting and planimetering of cross section data and the computation of volumes would be laborious and would consume an inordinate amount of time as far as the study was concerned. Therefore, arrangements were made with the Indiana State Highway Department to have the earthwork quantities computed on their IBM 650 electronic computer. The cross section data were transferred to computer punch cards for this operation, and the areas and volumes were determined quickly and accurately with the use of a standard earthwork program.

The California studies indicated that the accuracy of photogrammetric earthwork quantities could be considerably improved by adjusting cross section elevation readings to an accurate centerline profile (3,5). It was decided that this procedure should be included in the study along with the method of determining earthwork quantities without adjustments. Accordingly, all template sections were raised or lowered by an amount equal to the error at the centerline station, and the resulting earthwork quantities were compared with quantities determined by normal field methods.

The Indiana State Highway Department Construction Record for Project F-616 (3,5) was used in making comparisons, except in the case of embankment quantities, which were not listed in the construction record. Embankment quantities for comparison with the same photogrammetric quantities were computed from data obtained from the field cross section books. Excavation quantities were also compared in this manner in addition to the construction record comparisons.

For purposes of comparison, the reinforced concrete pavement was subdivided into twenty-one sections corresponding to the subdivisions of the construction record. Concrete pavement appurtenances (i.e., side road entrances and widened sections) were included within these subdivisions. Eight concrete private drive entrances within the city limits were treated separately, however. Fifteen sections of paved side ditch, eleven sections of guard rail, and twelve guide post groupings were measured and compared in correspondence with the quantity subdivisions of the construction record. Sodding was measured and compared on the basis of area quantities between 100 foot stations along the highway.

EQUIPMENT

Aerial Camera

A precision aerial camera of distortion-free or nearly distortion-free characteristics is required in the initiation of an aerial survey. For highway surveys, cameras of six-inch focal length and cycling times not in excess of two to three seconds are usually specified. The short focal length with its wide angular field yields better vertical mapping accuracy and gives complete coverage with fewer photos. The fast cycling time permits lower flying altitudes for large scale photography and higher aircraft speeds for more stable flight.

A modified K-17C government surplus aerial camera was used by the Indiana State Highway Department to obtain photography for the study section. The Bausch and Lomb Metrogon Lens used in the camera has a nominal focal length of six inches and a maximum aperture of f/6.3. The camera has a minimum re-cycle time of 1 1/4 seconds.

A report by the National Bureau of Standards on the distortion characteristics and resolving power of the camera is summarized in Tables 1 and 2. Table 1 gives the image displacements in millimeters from distortion-free positions at 7.5° intervals from the center of the plate. The probable error for these values does not exceed ± .02 millimeters. The values of resolving power at 7.5° intervals from the center of the field are shown in Table 2. These values were obtained by photographing test charts comprised of patterns of parallel lines. The row marked "tangential" gives the largest number of lines per millimeter in the negative image that can be distinctly resolved into separate lines when the lines lie perpendicular to the radius drawn from the center of the field. Similar values for lines lying parallel to the radius are given in the row marked "radial".

Table 1

Distortion Characteristics
(Millimeters)

Interval	7.5°	15°	22.5°	30°	37.5°
Displacement	0.00	0.01	0.05	0.07	0.05

Table 2

Resolving Power
(Lines per millimeter)

Interval	0°	7.5°	15°	22.5°	30°	37.5°
Tangential	63	63	46	46	39	32
Radial	63	63	53	46	46	39

<u>Surveying Equipment</u>

A level net for vertical control was established with the use of a Zeiss-Opton Ni 2 Self-Leveling Level and

Philadelphia level rod. The Zeiss level automatically sets its own line of sight level by means of a compensating pendulum prism. Leveling operations are greatly expedited since it is only necessary to center a circular spirit level approximately.

A Wild T2 theodolite, a subtense-bar, and a stadia rod were used to set up a traverse for horizontal control. The distance between points on the main line of the traverse was established by using the theodolite and subtense-bar; side shots were made by sighting on the stadia rod.

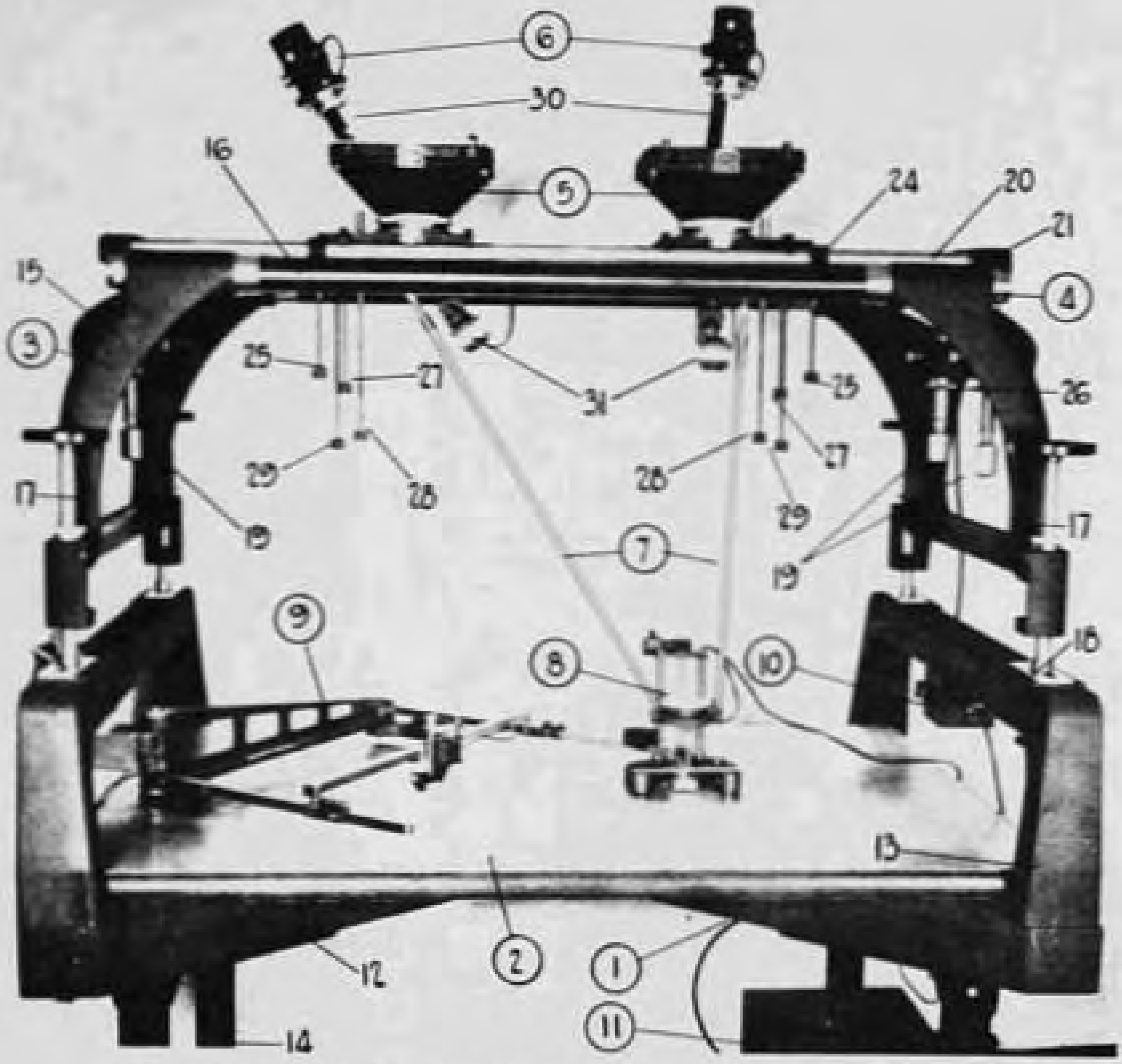
The Wild T2 differs from conventional transits in that both the horizontal and vertical circles are seen in an auxiliary reading telescope situated immediately adjacent to the main telescope. An internal system of lenses and prisms makes it possible to see the two opposite positions of each graduated circle simultaneously. The Wild T2 is commonly used for second and third order triangulations and other surveying operations which require high angular accuracy. The instrument can be read directly to one second of arc.

The subtense-bar may be used with the Wild T2 theodolite for indirect measurement of distances. The bar is mounted horizontally on a tripod and at right angles to the line of sight of the theodolite. Targets located at each end of the bar are separated by a distance of exactly two meters. With the aid of subtense distance tables or simple trigonometry, the angular distance between the targets as observed with the theodolite can be converted to the horizontal distance between the theodolite and the subtense-bar. Where high accuracy is desirable, the angle can be repeatedly measured and the error diminished as desired. Unlike direct stadia distance measurements, the distance obtained is a true horizontal distance and requires no slope correction. In essence, the subtense-bar is a portable, short base on which angles are read by a theodolite. It is of great value in making ground control surveys for photogrammetric mapping.

Kelsh Plotter

The standard Kelsh plotter used in this study is similar in principle to other types of double projection plotters in that the same general methods for projecting, viewing, and measuring the stereoscopic model are employed. Like other plotters in its class, it is capable of third order work. Unlike the multiplex plotter, the single model Kelsh cannot be used to bridge control from model to model. This, however, is not a serious limitation in highway work since adequate control by ground survey methods is usually available. In fact, the economy and ease of operation of the single model design, together with other features peculiar only to the Kelsh, have made it one of the more popular plotting instruments in highway work.

The Kelsh plotter and its components are shown in Plate II. The main components of the plotter are: (1) the table frame, (2) the table top, (3) the supporting frame, (4) the



(CIRCLED NUMBERS REFER TO MAIN ASSEMBLIES ONLY)

(1) TABLE FRAME
(2) TABLE TOP
(3) SUPPORTING FRAME
(4) PROJECTOR - TRACK FRAME
(5) PROJECTOR ASSEMBLIES
(6) ILLUMINATING UNITS
(7) GUIDE RODS
(8) TRACING TABLE
(9) PANTOGRAPH
(10,11) ELECTRICAL ACCESSORIES
(12) X - FRAME
(13) END BRACKETS
(14) TABLE FRAME SUPPORT
(15) END FRAMES
(16) SPACING BARS
(17) CLAMP SCREWS
(18) SLOTS
(19) LEVELING SCREWS
(20) PARALLEL BARS
(21) END CONNECTING BARS
(22) ADJUSTABLE FOOT SCREWS
(23) FOOT PLATES
(24) CLAMP SCREW
(25) FINE X - ADJUSTMENT
(26) LOCK NUTS
(27) SWING ADJUSTMENT
(28) X - TILT ADJUSTMENT
(29) Y - TILT ADJUSTMENT
(30) LAMP YOKE
(31) COUNTERWEIGHT BLOCK

KELSH PLOTTER AND COMPONENT PARTS

PLATE II

projector track frame, (5) two projector assemblies, (6) two illuminating units, (7) two guide rods, (8) the tracing table, (9) the pantograph, and (10,11) electrical accessories. The pantograph was not used in this study.

The projection system of the Kelsh has three unique features: the use of contact size dispositives; moving illumination units that concentrate light on a small portion of the model; and a cam arrangement for adjusting the principal distance of the projectors to compensate radial lens distortions. The contact-size glass diapositives, which are seated on top of the projectors, have the same format as standard 9x9-inch air-photo negatives. The image of the diapositives is magnified five times in the process of projection. Direct projection of the entire 9x9-inch image would require bulky and impractical equipment. The use of contact-size diapositives is made possible by the "swinging" illuminating units which scan the diapositive with a small intense circle of light. The illuminating units, generally termed lamphouses, are connected by guide rods to the tracing table in such a manner that the image is always directed toward the platen of the tracing table, regardless of the position of the tracing table. The principal distances of the projectors can be lengthened or shortened to compensate the distortion at each differential portion of the diapositive as it is scanned. The desired variation in the principal distance is determined by the optical systems of the projection apparatus and the aerial camera. This variation is accomplished by a small aspheric ball cam which is designed to a shape that will, by mechanical linkage to the lamphouses and the tracing table, raise or lower the projector lens by predetermined amounts for the various angles through which the cam can be rotated.

The projectors themselves are designed to simulate the taking cameras. The projector's focal length is adjusted to duplicate that of the camera, and for best results, the projector's lens should be identical to the camera's. The projectors are mounted so that they can be adjusted to duplicate the spatial relationship that existed between camera stations at successive exposures along the flight line. These adjustments as shown in Plate II are called (27) the swing adjustment, (22) the x-tilt adjustment, and (29) the Y-tilt adjustment. The projectors used in this study were equiped with a Y-adjustment to allow elimination of small amounts of local Y-parallax.

The viewing system of the Kelsh plotter is based on the principle of the anaglyph. Narrow beams of monochromatic light, one red and one blue, are projected through the diapositives and projector lenses to the platen of the tracing table. The platen, a white reflecting disk about 3 1/2 inches in diameter, is raised or lowered to focus the two independent images. By viewing the images through glasses with lenses corresponding to the complimentary beams of light, the observer perceives a three-dimensional model. The tracing table can be moved to observe any portion of the model. The intensity of each beam of light can be adjusted to different viewing positions by means of a rheostat.

A "floating mark" built into the center of the platen in the form of a small point of light provides the means of measurement within the model. Vertical motion of the floating mark is obtained by raising or lowering the platen. This motion is converted to differences in elevation in feet by a height-indicating scale. To determine the elevation of a point in the model, the observer adjusts the position of the platen so that the floating mark is in apparent contact with the point and then reads the elevation from the scale. All horizontal measurements are fixed orthographically. Horizontal motions of the tracing table can be recorded by means of a pencil mounted vertically below the floating mark.

The performance of the cams in compensating for radial lens distortion is one of the most important factors affecting the accuracy with which the optical model can be observed and measured. Before actual plotting was begun, the centering of the cam follower was tested by accepted laboratory procedures, and the performance of the cam was then checked by drawing the cam compensation pattern which resulted from the measurement of a stereoscopic model formed by a pair of accurate grid diapositives. The grid diapositives do not contain the displacements introduced to the photography by the distortion inherent in the metrogon camera lenses; therefore, the anticipated pattern of the surface of the grid model is the reverse of the deformation that would be expected from uncompensated metrogon photography.

A comparison of this pattern with the desired compensation pattern for a nominal metrogon lens showed fairly good agreement. Moderate departures from the desired pattern indicated that the cam followers were not precisely centered. It was not possible to affect a more accurate calibration of the cams, however.

IBM 650

The IBM 650 is a stored-program, modified single-address, numerical, decimal machine. It is made up of three units: the read-punch unit, the console unit, and the power unit. Input and output in the form of punch cards is through the read-punch unit. Computation and storage is accomplished by the calculating units and the rotating magnetic drum of the console unit. The power unit supplies the power for all machine units and contains the circuitry for translating the decimal input into machine code and vice-versa.

As used in connection with this study, the earthwork program punch cards are fed into the read-punch unit and the program instructions are stored on the surface of the magnetic drum in the form of magnetic spot patterns. Cross section data are fed in next. As computation takes place, the answers are stored on the drum. When computation is completed, the machine punches the results on output cards. These are placed in a tabulator where the information on them is printed in tabular form.

PROCEDURE

Photography

Procurement by the Indiana State Highway Department of suitable aerial photography and printing of the corresponding diapositives was the first step after the study site had been selected. The photography was flown at an altitude of 1,500 feet above average ground elevation. This altitude provided a contact scale of 1 inch = 250 feet and a manuscript plotting scale of 1 inch = 50 feet. The diapositives were made on .06-inch sensitized glass plates printed emulsion side up. The photography and diapositives were, in general, of high quality. The corners of some photos were slightly under-exposed, but as most cross section plotting was near the center of the photos, only the observation of elevation control points in the darker corners was adversely affected. Crab and drift caused a loss of the 50 percent overlap needed for stereoscopic coverage in one small area. This area was not within the limits of highway construction and so did not interfere with plotting operations. The crab and drift did cause some difficulty in the orientation of models, however.

Field Control

Elevation control points must be determined for at least three well spaced points in each stereo-model to define the plane of the model with respect to the datum, and at least two horizontal control points are needed to establish the scale and azimuthal orientation of the model. More than the minimum number of control points is desirable for checking purposes. Photo-points (i.e., horizontal and vertical ground control points) must be identifiable from the ground as well as on the photos, and they must be accessible from the ground.

The selection of photo-point control was based on a study of the air-photos of the study area. An attempt was made to select at least four vertical control points, one in each corner of the model, and two to three horizontal control points per model. This was not always possible since easily identifiable points in the desired places were sometimes non-existent (e.g., in a cultivated field or thickly wooded area). In all, 43 vertical control photo-points and 28 horizontal control photo-points were selected and later surveyed in the field. Figure 1 shows the positions of the photo-points selected in relation to the nine models which were plotted.

A series of level circuits tied into highway construction bench marks were used to determine the elevation of vertical control photo-points to .01 foot. An open subtense-bar traverse along the edge of the highway established the relative positions of horizontal control photo-points in

PHOTO – POINT CONTROL

FIGURE 1

terms of distance and deflection angles. The distance measurements for horizontal control points away from the main line of traverse were made by the stadia method to the nearest foot. The deflection and subtense-bar angles of the traverse were measured to one second or arc. The angular measurements were repeated three times and averaged at each instrument set-up to increase the accuracy of long shots and provide a check on instrument readings. A three-man party was used for both leveling and traversing.

Plotting Control Data

A continuous strip of tracing cloth, 18 feet in length and 40 inches in width, was used for the photogrammetric map manuscript. Distance and angular measurements were converted to coordinates of latitude and departure for convenience in fixing the position of horizontal control points on the manuscript. A north-south base line was ruled on the manuscript and the positions of the horizontal control points were plotted as accurately as possible by scaling the latitude and departure of each point from the base line. The base line was positioned on the manuscript in such a manner that the features of the highway were near the center of each model where the image is sharpest and distortion compensation is more reliable. The positions of elevation control points were not established until after individual models were set up in the plotter. Since the scale and azimuthal orientation of each model is fixed by the horizontal control points, the horizontal positions of vertical control points can be accurately determined from the orthographic projection of the points in the model.

Photogrammetric Plotting

Perhaps the most important and most time consuming phase of the study was the photogrammetric plotting of the final pay quantities. Because of the relative inexperience of the author with photogrammetric plotting equipment and the high precision of measurement desired by the study, extreme care had to be exercised in the orientation procedure and the measurement of quantities. Approximately 15 percent of the 224 hours of plotting time was consumed in the orientation phase. During plotting, repeated measurements, both horizontal and vertical, were often made at points where the true nature of construction and terrain was in doubt.

An instrument orientation procedure to reconstruct the same perspective conditions that existed between photographs at successive exposure stations and to relate the horizontal and vertical position of the model to the horizontal and vertical datum must precede the actual plotting of each model.

The proper perspective condition is achieved by "relative orientation", a systematic procedure of rotational movements of each projector until the conjugate images of the model are made to coincide over the entire model area.

This procedure of "clearing y-parallax" was accomplished by the "swing-swing" method of relative orientation in which the y-parallax was observed successively at each of six points in the model and cleared by making the proper swing and tilt adjustments of the projectors for each point observed. It was not always possible to clear y-parallax completely. In such cases, an attempt was made to distribute the parallax evenly over the entire model area.

The model formed by the completion of relative orientation has an undetermined scale and undetermined relationship to the horizontal and vertical control. To achieve "absolute orientation," the model is brought to scale by moving the projectors together or apart along the projector track frame so that the distance between model photo-points is made equal to the distance between their corresponding plotted control points on the manuscript. At the same time, the manuscript is adjusted in its horizontal plane to obtain an orthographic relationship between the horizontal control photo-points in the model and their pre-plotted positions on the manuscript.

The model is brought into agreement with the vertical control by a procedure called "leveling". An "index", or reference elevation, is established by first placing the floating mark in apparent contact with the spatial position of any one of the vertical control points of the model and then setting the vertical reading of the height counter to agree with the known elevation of that point. Vertical readings of other control points in the model indicate, by the amount of their failure to agree with known elevations, how the model must be leveled to agree with the vertical control. The model is leveled to the proper position by adjusting the support screws of the projector track frame.

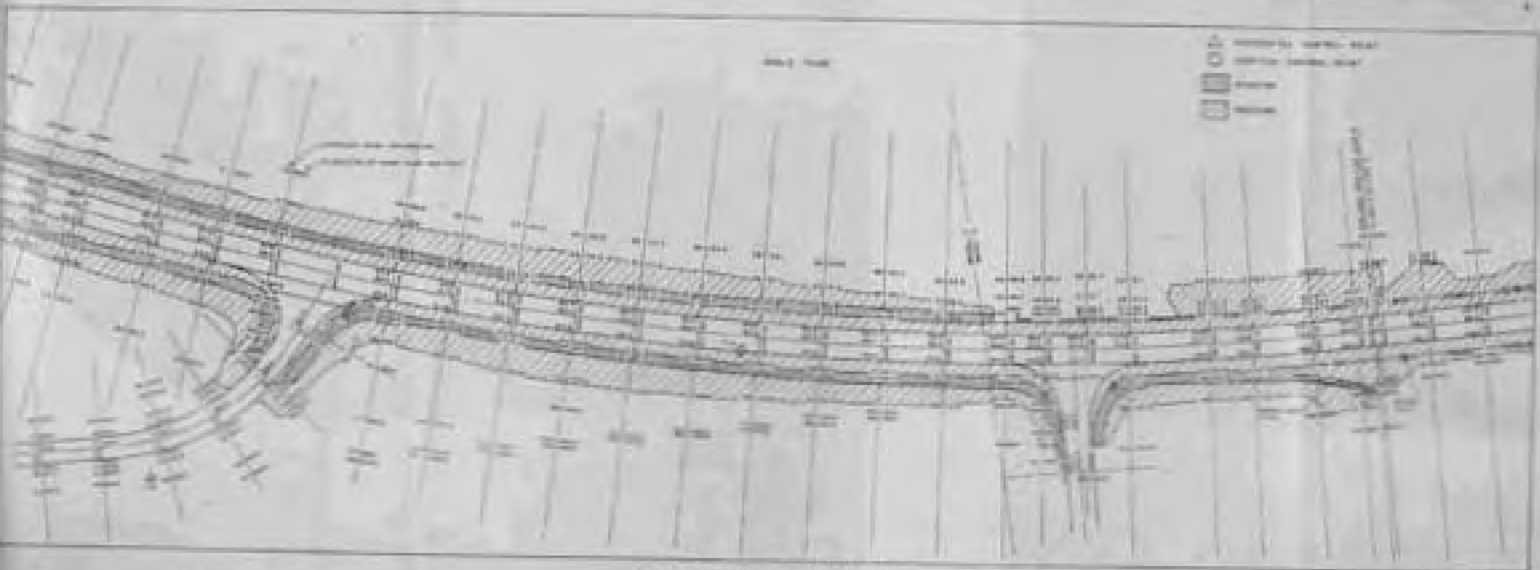
Not all of the vertical control points could be made to agree with their known elevations. In such cases the discrepancies in elevation, usually not more than a few tenths of a foot, were distributed as evenly as possible among all of the vertical control points of the model.

After the absolute orientation of each model had been completed, the actual plotting of final pay quantities was begun. Figure 2 illustrates the manner in which the quantities were plotted on the manuscript.

The planimetric outline of the pavement was first drawn on the manuscript by plotting points about one inch apart to delineate the centerline and edges. These points were then connected by using a straight edge or highway curves on the curved portions.

Cross section lines were drawn at right angles to the centerline at the proper centerline stations as indicated by the highway level books. Correct positioning of the cross section stations was achieved by referencing permanent station marks, which were stamped on the pavement at 500 foot intervals, to the horizontal ground control.

Spot elevations were read to 0.1 foot along the cross section lines at the centerline station, pavement edges, and at significant changes in the profile of the cross section line. The elevation readings were extended far enough



DETAIL OF PORTION OF MANUSCRIPT

FIGURE 2

on either side of the centerline to include any terrain which indicated earthwork movements. The position of the reading point was marked on the cross section line, and the elevation and the scaled distance of the point from the centerline were recorded on the right and left of the point respectively as shown in Figure 2.

Special borrow pits were plotted in the same manner, except that the cross section lines were referenced to a base line rather than the centerline of the highway.

The planimetric features of other final pay items were delineated on the manuscript by guiding the tracing table so that the floating mark and plotting pencil followed the feature being compiled. During this plotting operation, the floating mark was kept in apparent contact with the feature being compiled so that a true orthographic projection would result. The dimensions of these items were not recorded on the manuscript at the time of plotting but were later measured and tabulated upon the completion of each model. Large irregular-shaped areas, such as sodding, were planimetered several times and averaged to increase the accuracy of measurement. Areas of more constant dimensions were subdivided into simple geometric shapes for scaling and computation. Paved side ditch, guard rail and curbing were measured in the linear direction only. Guide posts were outlined and counted.

The engineer's scale used for making horizontal measurements on the manuscript had 50 divisions to the inch, thus allowing direct measurement to one foot and estimation to the nearest 0.5 foot. The horizontal measurements for locating the position of cross section elevations were made to the nearest foot while other horizontal measurements were made to the nearest 0.5 foot.

Organization of Data

To facilitate card punching for computer input, the cross section data were tabulated in the form used by the Indiana State Highway Department for presentation of earthwork data. This form with a sample tabulation of cross section data from the manuscript is shown in Figure 3. Elevations are placed above their corresponding horizontal coordinates in rows to the right of the station number and base elevation for each respective cross section. Any number of rows may be used to represent a single cross section as long as the data are arranged in order from left to right as it appears on the cross section line. The base elevation for each cross section line is determined so that no elevation reading exceeds three digits. If readings below the base elevation occur, they are subtracted from the base and given a minus designation. The earthwork program used for electronic computation required that the original terrain cross sections extend far enough on either side of the centerline to include the terminal elevation readings of the template sections. To conform with this requirement it was necessary to cut some template sections short by interpolation

EARTHWORK DATA

SQUAD CHIEF ____________
SECTION ____________ ROD READINGS CROSS SECTION NOTES DATE ____ PAGE 19 OF 61
LINE DESIGNATION F BOOK NO. ____ ELEVATIONS TEMPLATE NOTES ✓ RECORDER ____________

4	5-7	8	9-11	12-17	18-23	24	25-32	33-40	41-48	49-56	57-64	65-72	73-80
				480+00	900		(-3.0) / 50	96.7 / 42	95.9 / 28	96.5 / 12	96.8 / ℄	97.0 / 12	96.6 / 21
				480+00	900		95.9 / 26	99.7 / 46	99.7 / 76	- - -	- - -	- - -	- - -
				480+25	900		95.8 / 50	96.6 / 39	96.0 / 32	95.0 / 28	96.0 / 22	96.5 / 12	97.2 / ℄
				480+25	900		97.2 / 12	97.2 / 23	97.1 / 33	96.6 / 44	98.0 / 49	99.5 / 67	99.7 / 91
				480+65	1000		(-3.9) / 50	(-2.9) / 41	(-3.6) / 30	(-4.3) / 27	(-3.6) / 23	(-3.2) / 12	(-2.6) / ℄
				480+65	1000		(-2.3) / 12	(-2.3) / 24	0.3 / 93	- - -	- - -	- - -	- - -
				481+00	1000		(-2.8) / 50	(-2.6) / 38	(-3.1) / 31	(-4.2) / 27	(-3.2) / 22	(-2.8) / 12	(-2.4) / ℄
				481+00	1000		(-2.1) / 14	(-2.5) / 24	(-3.4) / 32	0.2 / 48	0.8 / 81	- - -	- - -
							- - -	- - -	- - -	- - -	- - -	- - -	- - -
IDENT		C	t_9-t_i	Station	H.I.	N	Y/X	Y/X	Y/X	Y/X	Y/X	Y/X	Y/X

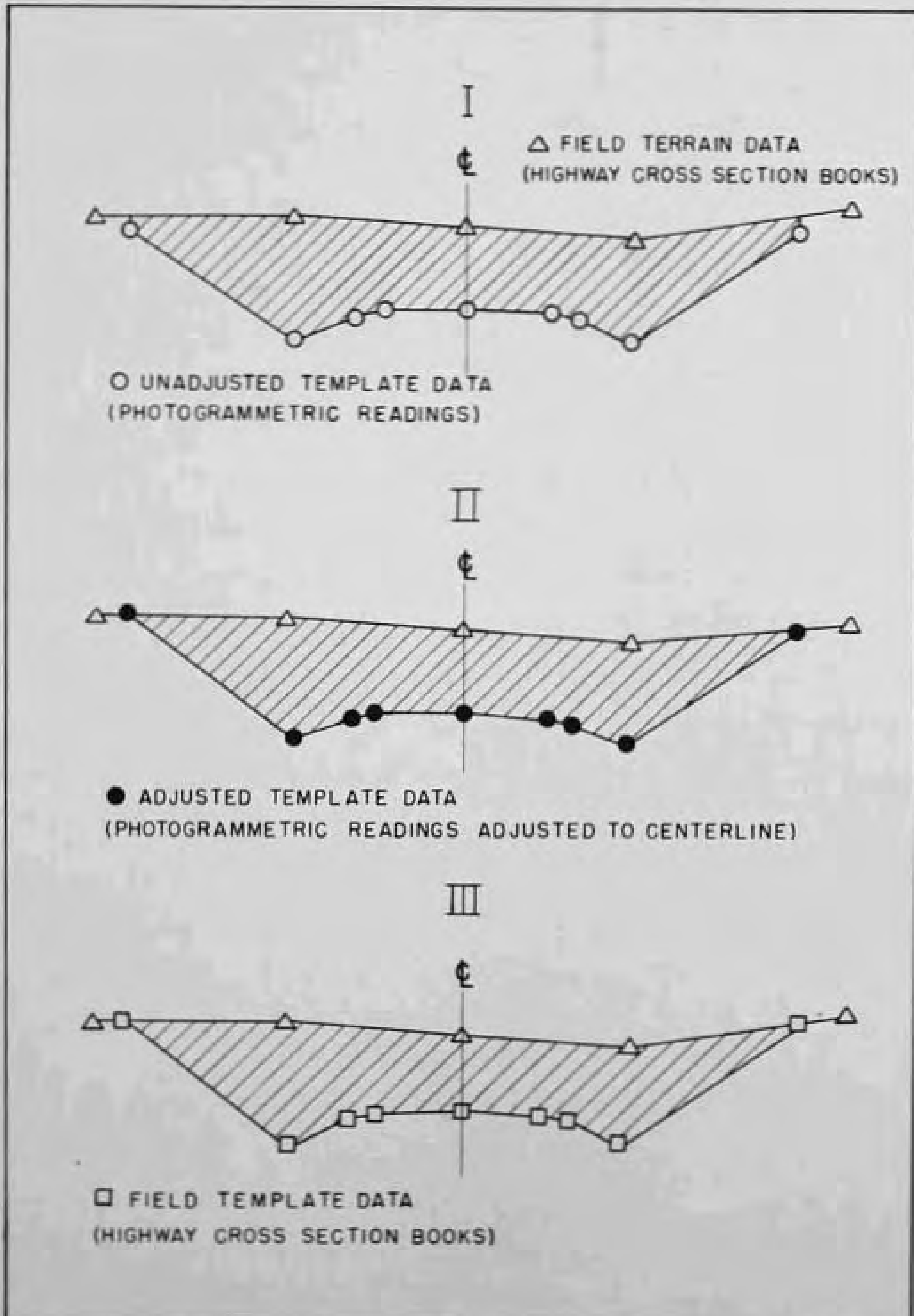
Cross Section Data Tabulation Form

Figure 3

between the template elevation readings on either side of the terminal terrain elevation.

Four earthwork data books, alike in format, were compiled from the manuscript data and the highway cross section books. The data book designated "Unadjusted Template Data" was compiled from the photogrammetric cross section data of the manuscript. The book designated "Adjusted Template Data" was compiled from the manuscript data by adjusting each cross section by an amount equal to the error at the centerline. The books designated "Field Terrain Data" and "Field Template Data" resulted from the conversion to elevation readings of the rod readings of the original and final field cross sections of the highway cross section books. Three separate sets of earthwork quantities were determined by combining each of the three template data books with the terrain data book. Figure 4 is a diagrammatic illustration of the cross sectional areas produced by combining these three sets of earthwork data.

The cross section data were broken down for computational purposes into four separate sections in each book: Line "F" (station 445+00 to station 483+03); Line "A" (station 480+15 to station 538+55); Line BP-3 (station 0+00 to station 2+76); and Line BP-4 (station 530+50 to station 536+00). The main line, or centerline, of the highway was divided into two sections, Line "F" and Line "A", by a station equation (station 483+03 Line "F" = station 480+15



CROSS SECTION DATA COMBINATIONS

FIGURE 4

Line "A"). BP-3 and BP-4 are borrow pit designations.

Before the non-earthwork quantities could be transferred to tables for comparison, the orthographic measurements from the manuscript had to be corrected for slope. For quantities such as curbing, pavement, guard rail, and paved side ditch where the slope was fairly constant, the correction was usually determined by the difference in elevation between the terminal points of the item measured. Where there were significant changes in slope between the terminal points, it was necessary to determine the slope correction for each segment which exhibited different slope characteristics. The procedure for determining slope correction for sodding was somewhat different in that a correction had to be applied to both the width and length of the sodded area. Since most sodding occurred in side ditches and on shoulder edges where the transverse slope profile was fairly constant (i.e., 2 to 1 or 4 to 1), a standard slope correction was applied to the transverse dimension and the longitudinal slope correction was determined in the manner described above.

Vertical Accuracy Study

In addition to the actual comparison of final pay quantities, a statistical analysis of vertical accuracy was undertaken. Vertical accuracy is perhaps one of the most critical considerations in the determination of earthwork quantities, and it is also an important factor in the determination of slope corrections for other quantities. There are no accepted standards by which to judge the vertical accuracy of spot elevation readings; however, the standards for vertical accuracy of contour mapping are well established and may be used as an indication of the precision with which spot elevations may be read. Certain statistical measures may also be used as an indication of precision.

The analysis was accomplished by a comparison of field centerline and base line elevations with corresponding photogrammetric elevations. The difference or "error" in the 239 elevation readings tested was treated as a random sample from the population of possible centerline and baseline elevation readings. The mean, variance, standard deviation, and range of the errors were computed, and the type and distribution of errors was investigated by tests on the means and variances of the individual models and on the variances within a single model.

ANALYSIS AND RESULTS

Vertical Accuracy Study

The variable under consideration in this analysis was the difference in elevation between the 239 centerline and baseline elevation points as measured by field survey methods and photogrammetric methods. These points were selected for comparison because both types of readings were made at identical locations along the centerline and baselines. It has been assumed for the purposes of this analysis that the elevation readings taken by field survey methods represent the true elevation values, and as such, may be used as a standard with which to compare the photogrammetric values. For this reason, the differences between the field survey readings and the photogrammetric readings are referred to as errors.

Inaccuracies in any system of measurement may be classified as random errors, systematic errors, and blunders. Random errors, or errors due to chance, can be expected to exhibit the distribution characteristics of the normal frequency distribution with a mean of zero and a symmetrical, bell-shaped distribution of errors on either side of the mean. The importance of a normal distribution of errors is emphasized by the fact that this distribution will tend to have a compensating effect on the total error in photogrammetric earthwork quantities. Systematic errors can generally be traced to assignable causes and corrective measures can be taken. Blunders in photogrammetric measurements can be reduced to a minimum by careful operation of the photogrammetric plotting equipment.

In the analysis of a large group of data, it is desirable to condense the data into a meaningful form which is characteristic of the entire group. A descriptive presentation of the elevation error data is given in Table 3 and Figures 5 and 6.

The frequency distribution tabulation of Table 3 shows the frequency of each error value, the per cent of the total for each error value, and the cumulative frequency per cent. The small range of errors in this distribution might be interpreted to mean that the photogrammetric readings were relatively free from large blunders.

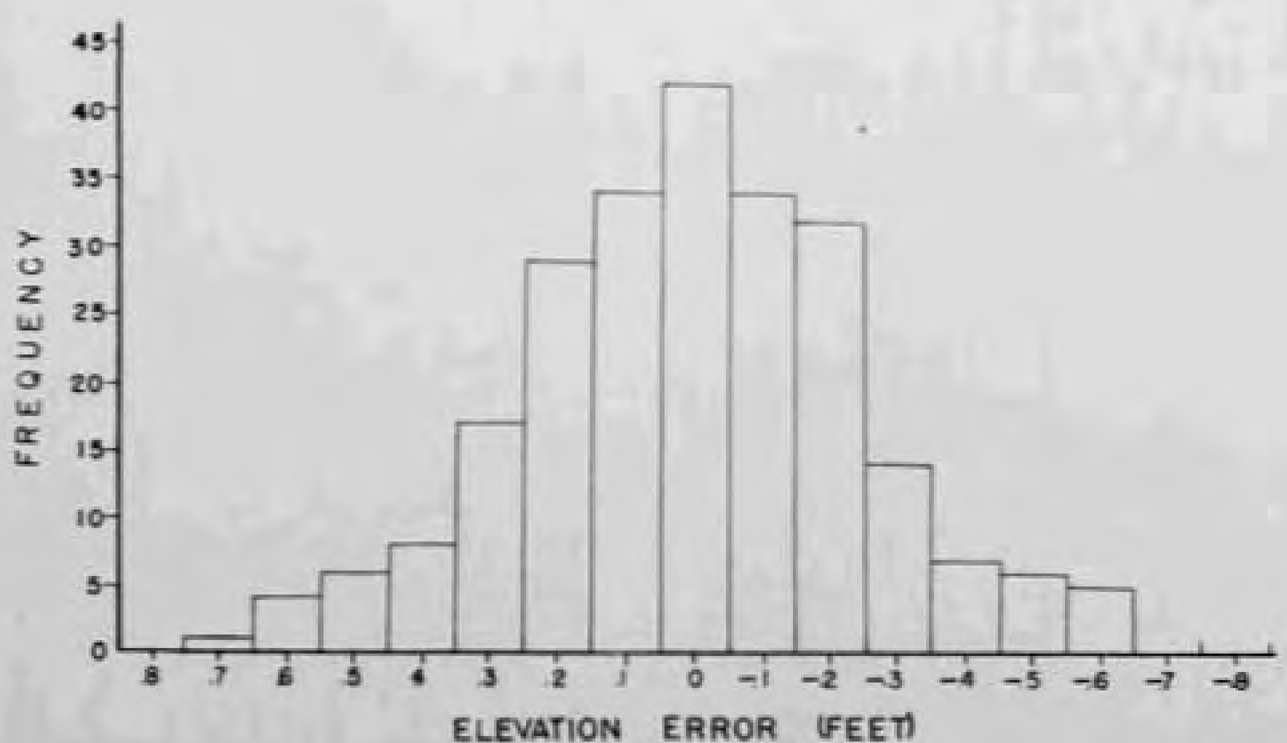
The data of Table 3 were used to plot the frequency polygon of Figure 5 which demonstrates the general conformity of the elevation errors to the symmetrical bell-shape of the normal curve.

Figure 6 is the same frequency distribution plotted in cumulative form with the abscissa scale graduated according to the area under a normal distribution curve. The points were plotted on the basis of cumulative per cent less than the class boundaries of the errors starting with the minus

TABLE 3

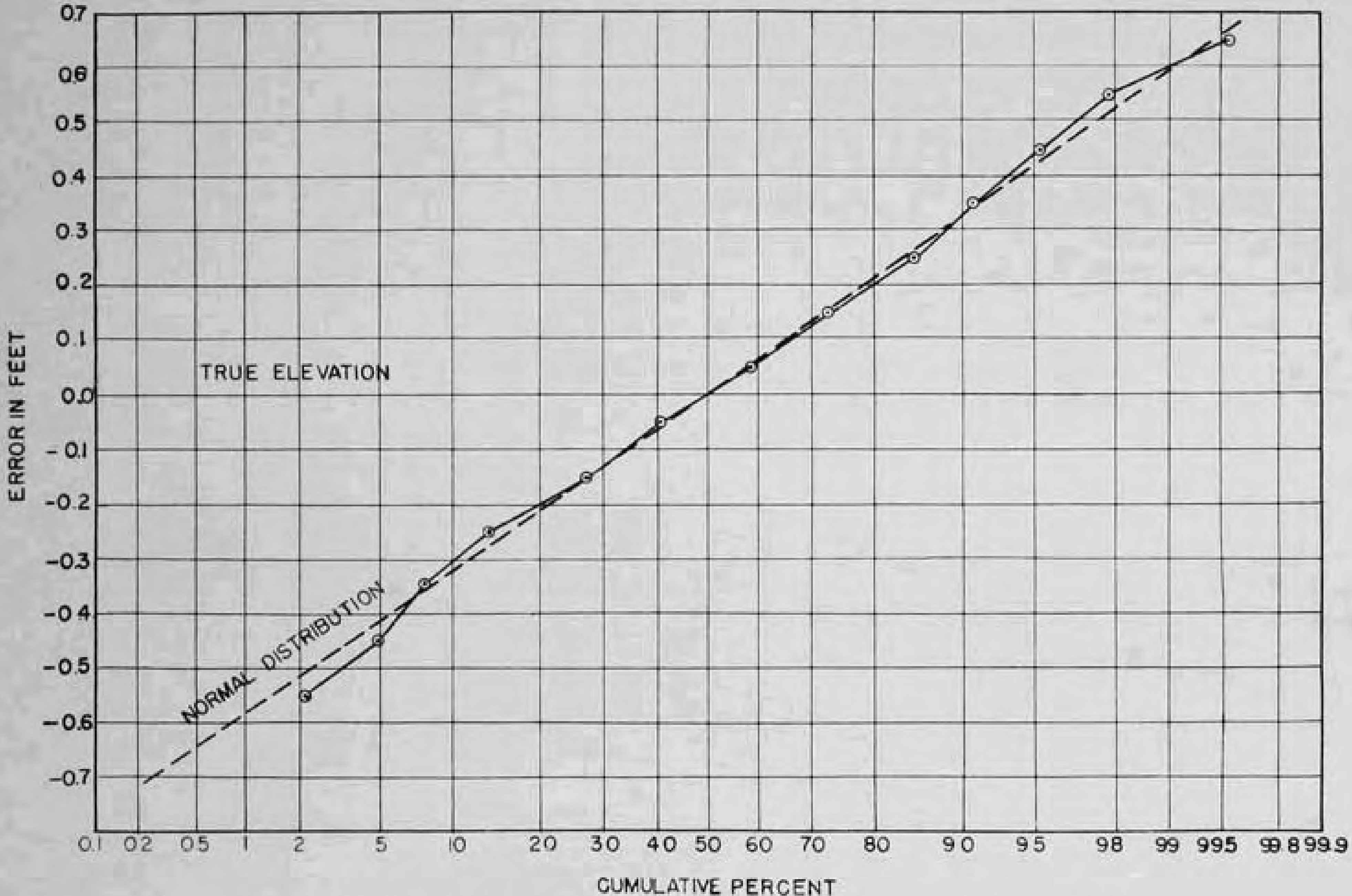
Frequency Tabulation of Elevation Errors

Error (Feet)	Frequency	% of Total	Cumulative %
- .6	5	2.1	2.1
- .5	6	2.5	4.6
- .4	7	2.9	7.5
- .3	14	5.9	13.4
- .2	32	13.4	26.8
- .1	34	14.2	41.0
0	42	17.6	58.6
.1	34	14.2	72.8
.2	29	12.1	84.9
.3	17	7.1	92.0
.4	8	3.4	95.4
.5	6	2.5	97.9
.6	4	1.7	99.6
.7	1	.4	100.0
Range 1.3	Total 239	Total 100.0	



FREQUENCY POLYGON OF ELEVATION ERRORS

FIGURE 5



CUMULATIVE FREQUENCY DISTRIBUTION OF ELEVATION ERRORS

FIGURE 6

values. The distribution of the elevation errors is in close agreement with the normal distribution function which is shown by a broken straight line.

The accuracy of photogrammetric plotting is frequently evaluated in terms of a C-factor which is defined as the flying height divided by the contour interval. The accuracy of the contour line and the magnitude of the contour interval is specified indirectly by the National Map Accuracy Standards which state that 90 per cent of the points tested must be within one-half contour interval of the true elevation (1). Although this study is concerned with spot elevation accuracy rather than contour line accuracy, an interesting comparison can be made between the accepted C-factor values for contour plotting and the C-factor calculated on the basis of the frequency distribution of the elevation errors. Kelsh plotter C-factors as used in large scale mapping range from 750 to 1500 depending upon the quality of the photography, plotting control, and other variables. However, a C-factor of 1200 is customarily employed for most plotting operations (5).

Approximately 90 per cent of the points tested fall within an error range of $\pm$ .45 feet as estimated from the cumulative frequency distribution of Figure 6. According to the National Map Accuracy Standards, this would allow a usable contour interval of 0.9 foot. At a flying height of 1500 feet, the calculated C-factor is approximately 1670, a comparatively high value.

Probably the first requirement for any method of measurement is that the dispersion or scatter of the measurements be small and that the central location of this dispersion be near the true value of the measurement. Where repeated measurements are made, they should be reproducible, i.e., repeated measurements of the same quantity should not differ too much from one another. The ability of a method of mensuration to produce measurements with a small dispersion about the true value is referred to as the precision of the method.

Where the central location of the measurements is not near the true values, systematic errors may be present. Typical of this type of error in measurement would be those due to incorrect distortion compensation or improper absolute orientation in the photogrammetric plotter.

The variance, standard deviation, and range are statistics which may be used to measure the dispersion of a group of measurements. The mean is a typical value which represents the location of the measurements as a whole. Since the precision of a method of measurement is determined by the dispersion and location of the results it produces, the above mentioned statistics are a convenient indication of the precision of the method.

The centerline and baseline elevation errors were processed to yield values of the mean, variance, standard deviation, and range for the entire project and for individual models and borrow pits. This subdivision of the data was used because each model was independent of all other models in the study. Differences in accuracy might be expected since each model was separately oriented and the quality of the diapositives varied from model to model. Separate model set-ups were used in the measurement of borrow pit quantities.

Table 4 lists the mean, variance, standard deviation, and range for the nine models and two borrow pits and for the entire project. The means for the individual models ranged from -.144 feet to .257 feet and the standard deviation ranged from .120 feet to .314 feet. The mean for the entire project was .009 feet with a standard deviation of .252 feet. The range in errors was 1.3 feet.

It is possible to draw statistical inferences from the data if certain qualifying assumptions are made. Since statistical inference is based upon the laws of probability it is necessary that the sample data be random, i.e., each measurement from the possible population of measurements has an equal chance of being selected for the sample. The elevation data for this analysis could not be fully treated as a random sample since only centerline and baseline points were included in the sample. However, it was thought that the data might be treated as a random sample from a population of measurements restricted to centerline and baseline elevations points only. The data were tested on this premise by the "theory of runs" and it was found that the 239 elevation

TABLE 4

Vertical Accuracy Measures

Model	No. Pts. Tested	Mean $\bar{x}$	Variance s^2	Std. Dev. s	Range
1	20	.165	.0984	.314	1.1
2	26	-.108	.0144	.120	.5
3	25	.188	.0629	.251	.9
4	23	-.117	.0377	.194	.8
5	16	-.144	.0267	.163	.6
6	32	.012	.0250	.158	.6
7	36	.089	.0331	.182	.6
8	20	-.220	.0974	.312	.9
9	23	-.030	.0213	.146	.9
BP-3	7	.257	.0633	.252	.6
BP-4	11	.155	.0750	.274	.8
Entire Project*	239	.009	.0635	.252	1.3

* All data treated as a single sample for computing variance and standard deviation.

$$s = \sqrt{\frac{n\sum x_i^2 - \sum(x_i)^2}{n(n-1)}}$$

errors generally exhibited random characteristics according to the criteria of the test (2). Even so, there was some question as to whether the sample was random since some points, by virtue of their location along the centerline and base lines, were more likely to be chosen than others. This of course, limits the value of any statistical inferences that might be made.

On the assumption that the data did constitute a random sample, tests were conducted under the hypothesis that the mean errors for the individual models and for the entire project were either equal or not equal to zero. These tests indicated that the mean error for the entire sample was not significantly different from zero at the five per cent level of significance* but that the mean errors for most of the models were significantly different from zero. Table 5 shows a tabulation of the tests on the means. Since the range of errors would indicate that no serious blunders were made, these tests would seem to indicate that systematic errors were operative in all models but model six and model nine where the means were not significantly different from zero. It appears that the systematic errors compensated each other in such a manner that the mean of all errors was practically zero.

The existence of systematic errors and the subsequent variation among the means can be more definitely established

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* The probability of an error in mean elevation as large or larger than that observed being due to chance alone is five per cent or less.

TABLE 5

Test of Means

HYP.: $\mu = 0$, ALT. HYP.: $\mu \neq 0$, $\alpha = .05$

Model	No. of Points Tested	t	$t_{n-1,.10}$	Significance
1	20	2.37	1.73	Significant
2	26	4.59	1.71	Significant
3	25	3.75	1.71	Significant
4	23	2.88	1.71	Significant
5	16	3.53	1.75	Significant
6	32	0.43	1.70	Not Significant
7	36	2.93	1.69	Significant
8	20	3.14	1.73	Significant
9	23	0.99	1.72	Not Significant
BP-3	7	2.69	1.89	Significant
BP-4	11	1.87	1.80	Significant
Entire Project	239	0.55	1.65	Not Significant

μ = Population mean

$t = \frac{\bar{x}-0}{s/\sqrt{n}}$

$t_{n-1,.10}$ = Percentage points of the Student t-Distribution for a one-tailed test at the five percent level of significance.

by a method known as analysis of variance (2). To carry out this analysis it is necessary to assume a model for the data which describes each observation. This model may be stated in equation form as follows: $x_{i\alpha} = \mu + \xi_i + \varepsilon_{i\alpha}$. Each observation $x_{i\alpha}$ is the sum of an overall mean μ, a systematic error ξ_i which represents the variation due to the class in which the observation occurs, and a random error $\varepsilon_{i\alpha}$ which represents the variation of any particular observation from the average value of the class in which it occurs. It is further assumed that each class is a sample from a normal distribution with mean $\mu + \xi_i$ and variance σ^2 and that the components of $\varepsilon_{i\alpha}$ are random variables. The two components of variance, ξ_i and $\varepsilon_{i\alpha}$, are estimated and a significance test is made. Table 6 shows the analysis of variance and the significance test at the five percent level of significance in which the data were classified according to plotter models and borrow pits. This test indicates that there was definitely an increase in the overall variation due to the different model classifications.

TABLE 6

Analysis of Variance

Model Assumed: $x_{i\alpha} = \mu + \xi_i + \varepsilon_{i\alpha}$ $\alpha = .05$

Source of Estimate	Sum of Squares	Degrees of Freedom	Mean Square
Between Classes s_i	4.36	10	.436
Within Classes $s_{\alpha(i)}$	10.76	228	.047
Total	15.12	238	

$F_{10,228.05} = 1.87$ $F_{10,228} = \frac{.436}{.047} = 9.24$ Highly Significant

The homogeneity of variation was examined by Bartlett's test and it was found that the variances of the eleven models were significantly different at the five percent level (2). The non-homogeneity of the variances and the existence of systematic errors would seem to indicate that the precision of elevation reading is dependent to a certain extent upon the individual model.

To test the precision of elevation readings within the model, repeated elevation readings were made at nine points selected at random from nine subdivisions of a single model area. Ten independent readings were made at each point. The variances were computed and analyzed by Bartlett's test. Again, the variances were found to be significantly different at the five per cent level.

The results of this test would seem to indicate that the variation in the precision of elevation readings is dependent not only upon the model but also upon the point at which the reading is made within the model. This would tend to coincide with actual plotting experience in which it was observed that the elevations of some points were more difficult to determine than others because of the varying degree of ground cover and image clarity and the lack of contrast in tone and terrain.

The data and analysis for this test and the preceding test are shown in Tables II and III of the Appendix.

Earthwork Quantities

Earthwork quantities were measured and compared on the basis of individual sections between succeeding cross sections and groupings of the individual sections by photogrammetric models and by various classifications of excavation and embankment. The adjusted and unadjusted photogrammetric quantities for the individual sections were compared with the construction record quantities and the field quantities which were computed electronically from the field cross section notes. Only excavation quantities were compared with the construction record.

Table I in the appendix shows the comparisons of the individual sections. The total excavation, exclusive of the borrow pits, was 161,856 yards for the construction record and 148,004 yards for the field quantities compared to 145,752 yards for the adjusted photogrammetric quantities and 143,524 yards for the unadjusted photogrammetric quantities. The adjusted and unadjusted photogrammetric quantities were in error from the construction record by -9.9 and -11.3 percent respectively, and they were in error from the field quantities by -2.0 and -3.5 percent respectively. The total embankment for the field quantities was 252,303 yards compared to 247,340 for the adjusted photogrammetric quantities and 246,002 yards for the unadjusted photogrammetric quantities for errors of -2.0 and -2.5 percent respectively.

The excavation quantities for the two borrow pits, also shown in Table I, yielded a somewhat better comparison between the photogrammetric quantities and the construction record quantities. The totals for Borrow Pit 3 were 13,260 yards for the construction record and 13,585 yards for the field quantities compared to 13,689 yards and 13,319 yards for the adjusted and unadjusted photogrammetric quantities. The adjusted and unadjusted photogrammetric quantities were in error from the construction record by 3.2 and 0.4 percent and from the field quantities by 0.8 and -2.0 percent respectively. The totals for Borrow Pit 4 were 26,861 and 30,077 yards for the construction record and the field quantities compared to 28,124 yards and 28,109 yards for the adjusted and unadjusted photogrammetric quantities. The adjusted and unadjusted photogrammetric quantities were in error from the construction record by 4.7 and 4.6 percent and they were each in error from the field quantities by -6.5 percent.

The poor comparison between the construction record and the field and photogrammetric quantities in excavation is due in large part to the shortened cross sections which had to be used in many places to allow machine computation of quantities as explained previously in the section on procedure. It would have been possible to extend the terrain cross sections to meet the slope stake points of the template cross sections so that all template readings would have been covered, but this involves an element of uncertainty as to the true nature of the terrain at the outer extremities of the cross sections. Since the field cross sectional areas were computed using the same shortened terrain data as the photogrammetric cross-sectional areas, it is felt that a better indication of photogrammetric accuracy and reliability is given by the comparison of field quantities and photogrammetric quantities.

The percentage errors used in making the comparisons of earthwork quantities may be very misleading if not interpreted with any consideration for the size of the quantities involved. This is especially true of very small quantities where an error of a few yards can result in a very large percentage error. In many instances where the quantities used as a basis of comparison were zero, the resulting percentage errors were incalculable. On the other hand, very large quantities, such as the total yardage for the entire project, may show a low percentage error and yet have a yardage error that might be considered undesirably large for pay purposes. Although the percentage error method has this inherent defect, it is the only method available for evaluating earthwork quantity comparisons.

Large percentage errors were generally associated with small earthwork quantities and, conversely, small percentage errors were generally associated with large earthwork quantities. Table 7 lists the number of individual earthwork sections from a total of 238 which were in error more than ten percent for quantities in excess of 100 and 200 yards.

There is a noticeable drop in the number of errors which were greater than ten percent for quantities in excess of 200 yards.

TABLE 7

Magnitude of Errors

Earthwork	Number of Sections in Error More Than Ten Percent			
	Unadj. and Const. Record	Adj. and Const. Record	Unadj. and Field	Adj. and Field
Greater than 100 yards	60	67	31	13
Greater than 200 yards	45	48	20	6

Although the adjusted photogrammetric quantities did not reduce the magnitude of the errors for the construction record comparisons, a considerable improvement resulted from adjustment in the field comparisons.

An examination of the individual sections of Table I will show that the photogrammetric quantities even after adjustment, were generally less in both excavation and embankment than the corresponding construction record and field quantities. In search of an explanation for this peculiar characteristic, it was observed that the positions of shoulder, ditch, and slope stake points were generally closer to the centerline for the photogrammetric determinations than for the same points as indicated by the field cross section books. This would account for a reduction in cross-sectional areas and a subsequent reduction in yardage. Since the horizontal positions of these points could have been in error for either the field or the photogrammetric determinations, it can only be speculated as to which is more nearly correct. However, there is sometimes a tendency to measure along the slope rather than horizontally when measuring the positions of rod readings in the field. This, of course, places the positions of the rod readings farther from the centerline than they actually are.

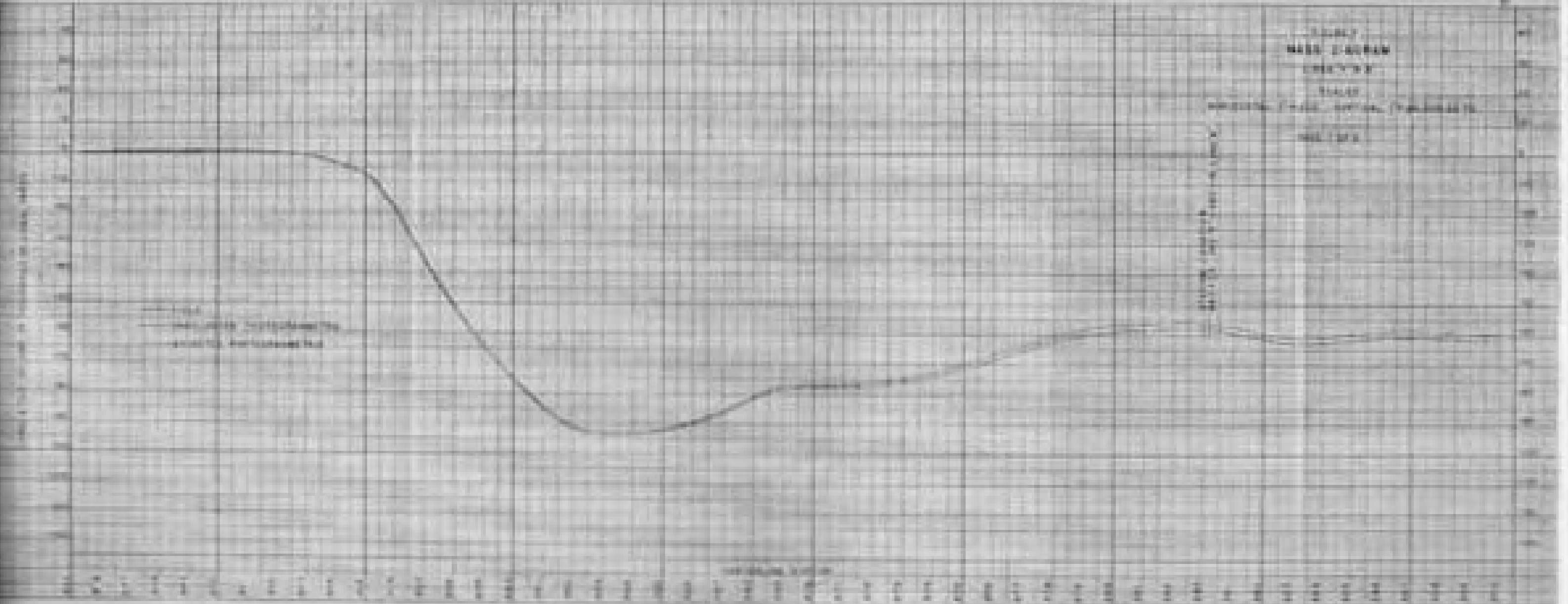
Further examination of the individual sections revealed that many of the adjusted quantities were in error more than the unadjusted quantities. In fact, the cumulative total over the entire project for the adjusted quantities was in greater error than the unadjusted quantities. Of the 197 cross sections which were adjusted, 56 were in the wrong direction (i.e., the cross-sectional area was reduced when it should have been increased and vice versa). Of the 141 adjustments in the right direction, 26 over-corrected and caused the error in cross-sectional area to be larger than the unadjusted value. In all, 82 of the adjustments actually reduced the accuracy of the earthwork quantities.

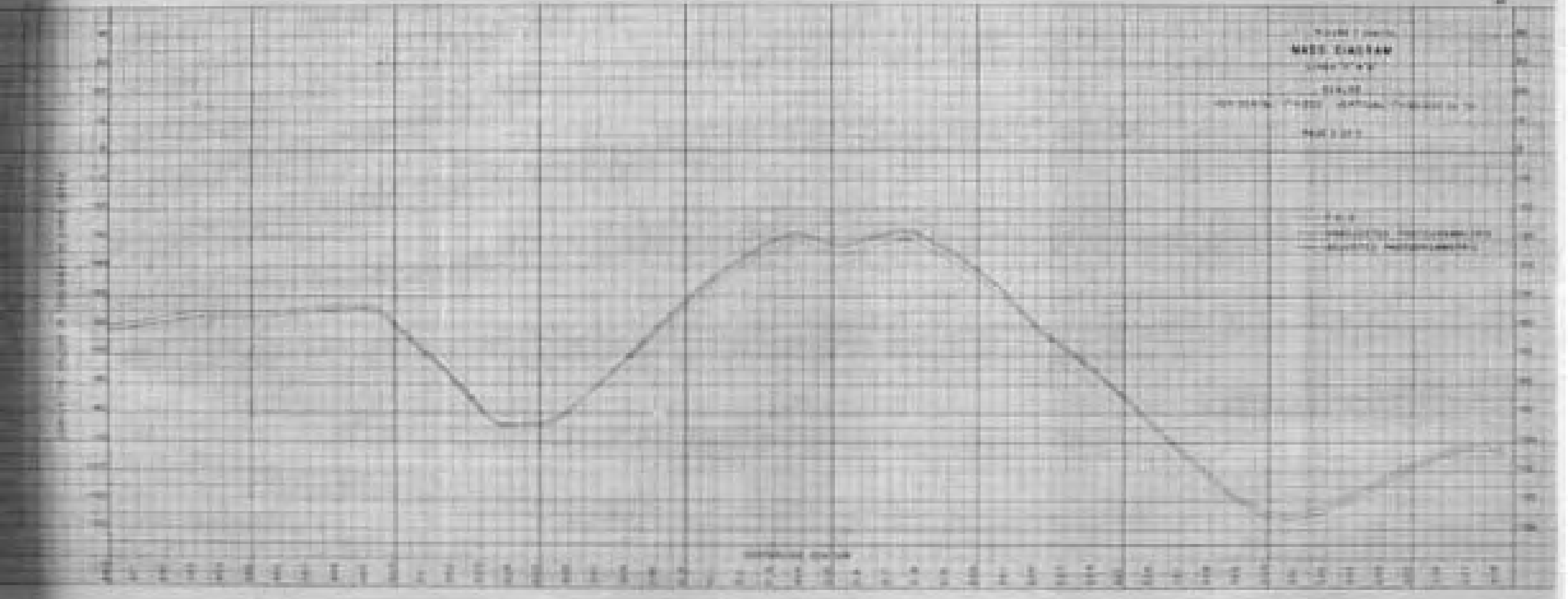
Immediately the question comes to mind as to whether or not centerline adjustments actually do improve earthwork quantities. By chance alone, half of the adjustments could be expected to be in the wrong direction and to show no improvement in the cross-sectional areas. To test the possibility that any improvement in direction or cross-sectional area was due to chance, it was hypothesized that the probability of no improvement was 0.5. For this test it was assumed that the adjustment data approximated a normal distribution.

It was found that the observed result of 56 adjustments in the wrong direction was 5.98 standard deviations lower than the expected value of 98.5 adjustments in the wrong direction. The probability of such an occurrence is for all practical purposes zero. The hypothesis that improvement in direction is due to chance should be rejected on the results of this test. The observed result of 82 adjustments showing no improvement in cross sectional area was 2.29 standard deviations lower than the expected value of 98.5 adjustments showing no improvement in cross-sectional area. The probability of this is .029 and at the five percent level of significance calls for rejection of the hypothesis that improvement in cross sectional area is due to chance.

Although the number of bad adjustments may appear to be large, these tests would seem to be a further indication that centerline adjustments definitely improve the accuracy of photogrammetric earthwork quantities.

The cumulative yardage for the field and photogrammetric quantities is shown by the mass diagram of Figure 7. The fairly close agreement between the field quantities and the photogrammetric quantities is illustrated by the close proximity of the three lines at most points. The greatest difference in cumulative yardage occurred at station 517+35 where the unadjusted yardage differed from the field yardage by 3,327 yards. The greatest difference between the adjusted yardage and the field yardage occurred at station 486+00


MASS DIAGRAM


MASS DIAGRAM

where the adjusted value differed from the field value by 1906 yards. Although the cumulative yardage at the end of the project showed the unadjusted cumulative yardage to be closer to the field value, the adjusted cumulative yardage was generally closer to the field value than the unadjusted for the greater portion of the mass diagram.

The cumulative yardage was the same at five points on the mass diagram for the field and unadjusted values and at two points for the field and adjusted values. Again, this demonstrates the effect of compensating errors on earthwork quantities.

The comparison of earthwork quantities by models to the field earthwork quantities showed more consistency in the percentage error range. These comparisons are shown in Table 8. Most of the errors for both the adjusted and unadjusted quantities were less than ten percent; however, Model 8 was in error by 800 percent in the comparison between the field and unadjusted photogrammetric excavation quantities. The percentage errors ranged from 0.9 percent for the unadjusted embankment quantities of Model 2 to 800 percent for the unadjusted excavation quantities of Model 8. The combined excavation and embankment errors ranged from -1.1 percent for the adjusted quantities of Model 7 to 13.2 percent for the unadjusted quantities of Model 5.

The yardage errors ranged from 16 yards for the adjusted embankment quantities of Model 3 to -4433 yards for the unadjusted embankment quantities of Model 8. Although the large

TABLE 8

Earthwork Comparison by Models (Cu. Yd.)
(Borrow Pits Excluded)

Model	Field Cut	Unadj. Photo. Cut.	% Error	Adj. Photo. Cut.	% Error	Field Fill	Unadj. Photo. Fill	% Error	Adj. Photo. Fill	% Error	Combined Error % Unadj.	Combined Error % Adj.
1	1,060	1,232	16.2	1,311	23.7	19,739	19,739	- 1.2	19,106	-3.2	- 2.2	- 4.7
2	7,803	7,672	- 1.7	7,411	- 5.0	75,003	75,659	0.9	76,176	1.6	1.2	2.3
3	28,824	27,097	- 6.0	27,812	- 3.5	298	538	80.5	314	5.4	- 6.9	- 3.6
4	4,096	4,075	- 0.5	3,787	- 7.5	5,619	5,405	- 3.8	5,507	-2.0	-12.7	12.9
5	6,038	6,363	5.4	6,096	1.0	1,895	1,672	-11.8	1,821	-3.9	13.2	3.2
6	12,105	12,310	1.7	12,243	1.1	43,392	41,939	- 3.3	41,822	-3.6	- 5.3	- 5.5
7	63,969	60,533	- 5.4	62,874	- 1.7	11,820	11,431	- 3.3	11,317	-4.3	- 5.8	- 1.1
8	13	117	800.0	96	638.5	89,298	84,865	- 5.0	86,282	-3.4	- 5.1	- 3.5
9	24,756	24,125	- 2.5	24,122	- 2.6	5,239	4,996	- 4.6	4,995	-4.7	- 2.0	- 2.0
Total	148,664	143,524	- 3.5	145,752	- 2.0	252,303	246,002	- 2.5	247,340	-2.0	- 1.1	- 2.0

yardage and percentage errors of Model 8 were associated with very large embankment quantities and small excavation quantities, it should be noted that the statistical measures of precision in the vertical direction as indicated in Table 4 were poor for this particular model. Visual inspection failed to reveal any correlation between the accuracy of the earthwork quantities of the other models and their corresponding statistical measures of vertical precision as shown in Table 4.

The reduction in error magnitude for the models in comparison to the individual sections suggests that compensating errors were operative to a limited extent. The relatively small errors that resulted from combining the excavation and embankment quantities further emphasizes the effect of compensating errors on earthwork quantities.

Seven groups of individual earthwork sections were classified according to various depths of excavations and embankments and compared with the corresponding field quantities. The selection of sections for each depth of classification was made on a relative basis by stereoscopic study of the air-photos for the entire length of the project. These classifications included shallow, medium, and deep cut and medium and deep fill. There was no group of sections of shallow fill large enough for consideration. Table 9 shows these comparisons along with an urban and rural classification of earthwork quantities.

TABLE 9

Earthwork Comparisons by Various Classifications
(Cu. Yd.)

Classification	Field	Unadj. Photo.	% Error	Adj. Photo.	% Error
Shallow Cut 445 to 451	785	946	20.5	912	16.2
Medium Cut 463+55 to 498+70	48,334	46,955	- 2.9	46,743	- 3.3
Medium Cut 529+80 to 537+00	24,254	23,631	- 2.6	23,597	- 2.7
Deep Cut 503+86 to 514	67,602	64,362	- 5.1	66,709	- 1.6
Medium Fill 517+35 to 529+80	98,431	93,577	- 4.9	95,088	- 3.4
Deep Fill 452 to 463+55	94,624	94,999	0.4	95,119	0.5
Deep Fill 498+70 to 503+86	41,571	40,386	- 2.9	40,320	- 3.0
Urban Cut 445 to 453+35	900	1,019	13.2	1,050	16.7
Rural Cut 453+35 to 538+20	147,764	142,505	- 3.6	144,702	- 2.1
Urban Fill 445 to 453+35	2,226	2,443	9.7	2,226	0.0
Rural Fill 453+35 to 538+20	250,077	243,559	- 2.6	245,114	- 2.0

The percentage errors for the various cut and fill classifications ranged from 20.5 percent for an unadjusted quantity of shallow cut to 0.4 percent for an unadjusted quantity of deep fill. The percentage errors generally followed an inverse relationship between the depth of the earthwork and the magnitude of the percentage error. The unadjusted quantity for one deep cut from station 503+56 to station 514+00 was in error more than earthwork quantities of medium and shallow cut, however.

The percentage errors gave no indication of any noticeable difference between the accuracy of excavation and embankment quantities. There was a noticeable difference between the different classifications according to depth, however. Only three of the seven classifications showed improvement after adjustment.

The percentage errors for the urban classification were greater than those for the rural except for the adjusted embankment quantities. The combined excavation and embankment errors were -11.3 and 7.4 percent for the adjusted and unadjusted quantities of the urban classification and -1.9 and -1.2 percent for the adjusted and unadjusted quantities of the rural classification.

It is doubtful that these differences can be attributed to the intrinsic characteristics of the urban and rural classifications. It is more likely that the difference in the depth and size of the earthwork quantities for each classification was of greater predominance.

In summary it is apparent that three important aspects of the use of photogrammetric earthwork quantities for final pay purposes have been developed in this study: (1) the effect of compensating errors; (2) the effect of centerline adjustments; and (3) the effect of various sizes of earthwork quantities.

The balancing of plus and minus errors has been demonstrated in both the vertical accuracy study and the earthwork quantity analysis. The range in vertical errors for the 239 elevation points tested was 1.3 feet with mean errors for the individual models of from -.220 feet to .257 feet. In contrast to the wide variation in errors for the individual points and models, the mean error for the total was only .009 feet, a value which was shown not to be significantly different from zero. Apparently the random errors, systematic errors, and blunders compensated in such a manner that the mean of all errors was practically zero. The effect of compensating errors was further demonstrated by Table I of the Appendix in which large errors for some of the individual sections were observed in both the plus and minus direction but in which the error for the entire project was relatively small. A graphic illustration of this compensating effect was provided by the mass diagram of Figure 7 in which the earthwork quantities were found to be identical at several points along the length of the project. The same compensating effect, but to a lesser degree, was observed in the grouping of quantities by models and by various classifications of excavations and embankments. It should be noted, however, that large differences in yardage, even though percentage errors were relatively small, existed over some lengths of the project. Apparently the compensation of plus and minus errors is affected over relatively long sections of earthwork quantities and can be relied upon only where considerable lengths of earthwork quantities are to be determined.

The probable existence of systematic errors established the need for centerline adjustments. These adjustments improved earthwork quantities on the overall basis even though some individual quantities and groupings of quantities showed a reduction in accuracy after adjustment. The possibility that any improvement in the accuracy of earthwork quantities could be attributed to chance was rejected on the results of statistical tests.

The percentage error appears to vary inversely with the depth and size of the section measured. This was demonstrated by comparison of various sizes of excavations and embankments in which it was observed that the percentage errors for shallow sections were larger than those for deeper sections. It was also observed that the number of errors in excess of ten percent was considerably reduced as the size of the earthwork quantity was increased from 100 to 200 yards. This type of variation in accuracy is to be expected since a given yardage error for a section in shallow cut might result in a very large percentage error while the same yardage error for a section of the same length in deep cut would result in a smaller percentage error. These observations would tend to favor the use of photogrammetric measurements on projects where large excavations and embankments predominate.

In evaluating the accuracy of photogrammetric methods, as applied to the measurement of final pay earthwork quantities, it must be recognized that there are no accepted tolerance limits for the accuracy of earthwork quantities and that the determination of earthwork quantities by any method is only approximate. Traditionally, earthwork quantities have been measured in the field and it has been assumed that these measurements are the exact dimensions. Volumes are computed from these measurements to the nearest yard or tenth of a yard. It is assumed that these volumes are correct; hence, no tolerance limits for accuracy are needed. A loose tape or a sloping level rod in the hands of a careless field crew would surely invalidate these assumptions. Even if the likelihood of field errors were considered, it is improbable that a reliable method for determining the accuracy of earthwork quantities could be established because of the difficulty and cost of obtaining a field survey of sufficient precision to use as a standard of comparison. In addition to the undetermined accuracy of field surveys, the actual terrain features can only be approximated by a few points which are thought to accurately represent the features of the terrain. As pointed out previously, it would take an infinite number of elevation readings to give a true representation.

Since no tolerance limits for the accuracy of earthwork quantities are available, it is difficult to state whether or not a certain error is within reasonable bounds. One might arbitrarily select an allowable error of say five percent. This is not unreasonable in light of the approximate methods used in determining earthwork quantities either photogrammetrically or by field surveys. In fact, it might be questionable as to which method is really more accurate.

Since payment to the contractor is made on the basis of excavation quantities only, special attention should be given to these quantities in any final pay considerations. The total excavation for the entire project, borrow pits included, was 192,320 yards for the field quantities compared to 187,565 yards for the adjusted photogrammetric quantities. The difference of 4,761 yards is in error by only 2.5 percent, or one half of the arbitrarily selected value of five percent. This is a very small error in view of the fact that either of the quantities might be in error. From the contractor's standpoint, a yardage error of this magnitude in the minus direction represents a considerable loss in payment for work done. It must be understood, however, that the difference could just as well have been in the contractor's favor. In any event, there is a good possibility that once the contractor understands the advantages and accuracies as well as the operational limitations of photogrammetric measurement of earthwork quantities, he will accept them as a basis for payment.

The results of this study are also an indication of the accuracy that might be achieved in the use of photogrammetric methods in the determination of design earthwork quantities. Since these quantities are used only as estimates, the accuracy required is not as great. Even the unadjusted quantities, which were in error by only 3.8 percent for the entire project, would surely be considered accurate enough for this purpose.

Concrete Pavement and Appurtenances

The photogrammetric measurements of concrete pavement and concrete pavement appurtenances were combined under one pay quantity classification and compared with the field measurements in accordance with the quantity subdivisions of the construction record. The pavement appurtenances consisted of widened sections to accomodate parked vehicles, street intersections, side road entrances, and a transition zone for change in lane width. In all, 21 sections of varying length and width were compared. Table 10 shows these comparisons.

The areas of sections of constant width were determined by simply multiplying the width times the length between the

TABLE 10

Reinforced Concrete Pavement Measurements

Station	Description	Photogrammetric Measurement (Sq. Yd.)	Field Measurement (Sq. Yd.)	Error (Sq. Yd.)	% Error
445+00					
447+85.8	Constant Width	1270.2	1270.2	0.0	0.0
448+50.2	Street Intersection	428.2	426.5	1.7	0.4
451+14	Constant Width	1172.4	1172.4	0.0	0.0
451+78	Street Intersection	375.4	381.6	- 6.2	- 1.6
453+20	Constant Width	629.3	629.3	0.0	0.0
454+90.7	Lane Width Transition	584.1	579.6	4.5	0.8
473+79.4	Constant Width	5036.5	5036.5	0.0	0.0
475+20.6	Road Intersection	491.8	510.6	-18.8	- 3.7
479+86.6	Constant Width	1242.7	1242.7	0.0	0.0
481+19.3	Road Intersection	452.6	453.2	- 0.6	- 0.1
483+03.3	Constant Width*	490.7	486.1	4.6	0.9
480+15	Station Equation	---	---	---	---
498+64	Constant Width	4930.7	4930.7	0.0	0.0
499+36	Road Intersection	249.3	250.7	- 1.4	- 0.6
521+17.6	Constant Width	5817.6	5817.6	0.0	0.0
521+82.4	Road Intersection	226.2	226.1	0.1	0.0
527+09	Constant Width*	1404.2	1392.5	11.7	0.8
529+37	Constant Width	608.8	608.8	0.0	0.0
530+03.5	Road Intersection	235.4	237.4	- 2.0	- 0.8
530+55.5	Constant Width	138.7	138.7	0.0	0.0
531+20.8	Road Intersection	235.0	230.8	4.2	1.8
538+55	Constant Width	1957.7	1957.7	0.0	0.0
Totals		27,977.5	27,979.7	- 2.2	0.0

* Discrepancy in area of constant width pavement due to chaining equation in field measurements.

terminal stations for each section. Since the centerline stationing on the photogrammetric manuscript corresponded to that used in making lineal field measurements, there was little chance for error once the width had been properly ascertained. Where discrepancies did occur, they were due to chaining equations in the field measurements. No slope corrections were applied because the grades were very small and such corrections would have been negligible. Areas at street intersections, side road entrances, and at the lane width transition were determined by subdividing the sections on the photogrammetric manuscript into simple geometric shapes and computing the areas from the scaled dimensions.

Most errors occurred where varying widths were encountered. The largest error, -18.8 square yards or -3.7 percent, was made in determining the area at a street intersection. All errors were very small; 18 of the 21 sections were in error less than one percent. The total area by field measurements was 27,979.7 square yards compared to 27,977.5 square yards by the photogrammetric measurements for an error of -2.2 square yards, or a percentage error of practically zero. As with the earthwork quantities, the plus and minus errors over the entire project appear to have compensated and, in this case, made the total error almost negligible.

Separate comparisons of eight private drive entrances, also regarded as pavement appurtenances but with a different pay quantity classification, similarly showed good agreement

TABLE 11

Private Drive Concrete Pavement Measurements

Approximate Station	Photogrammetric Measurement (Sq. Yd.)	Field Measurement (Sq. Yd.)	Error Sq. Yd.	% Error
446+53.6	13.3	14.5	-1.2	- 8.3
446+71.3	34.8	34.8	0.0	0.0
448+66	13.1	13.2	-0.1	- 0.8
448+95	17.3	16.9	0.4	2.4
449+21	11.2	12.0	-0.8	- 6.7
450+36	11.6	11.7	-0.1	- 0.9
450+56	13.4	14.5	-1.1	- 7.6
451+32	7.0	6.9	0.1	1.5
Totals	121.7	124.5	-2.8	- 2.2

between the photogrammetric and field measurements. These comparisons are shown in Table 11.

The errors ranged from -1.2 square yards, or -8.3 percent, to zero. Again, some compensation of errors was evident, but to a lesser extent than with the other paved surfaces. The total area as computed from the field measurements was 124.5 square yards compared to 121.7 square yards for the photogrammetric measurements for an error of -2.8 square yards, or -2.2 percent.

The results of these comparisons would seem to indicate that fairly accurate measurement of pavement and pavement appurtenances can be obtained photogrammetrically.

Paved Side Ditch

Photogrammetric measurements of paved side ditch were compared with the field measurements in accordance with the quantity subdivisions of the construction record. Since payment to the contractor is made on a lineal foot basis, measurements were made along the length of the paved side ditch only. All measurements were corrected for slope. Table 12 shows the comparisons between the photogrammetric and the field measurements as listed in the construction record.

Errors in identification as well as in measurement were encountered in the determination of paved side ditch lengths. The construction record listed 15 sections of paved side ditch; but, due to snow cover, only 13 were observed and

TABLE 12

Paved Side Ditch Measurements

Approx. Station Interval	Photogrammetric Measurement (Feet)	Field Measurement (Feet)	Error (Feet)	% Error
453+35 to 456+77	396.0	396.0	0.0	0.0
456+97 to 463+88	698.0	654.5	43.5	6.6
469+30 to 469+90	Snow Cover	62.0	-62.0	-100.0
482+15 to 482+85	67.5	69.5	- 2.0	- 2.9
487+75 to 488+65	131.5	132.0	- 0.5	- 0.4
493+42 to 494+42	102.0	101.0	1.0	1.0
498+24 to 499+60	162.5	166.0	- 3.5	- 2.1
499+68 to 500+31	62.0	62.0	0.0	0.0
500+36 to 500+69	Snow Cover	34.0	-34.0	-100.0
513+34 to 514+42	167.5	171.0	- 3.5	2.0
516+98 to 517+50	83.0	92.0	- 9.0	- 9.8
522+52 to 522+89	36.0	38.0	- 2.0	- 5.3
523+00 to 525+70	304.0	309.0	- 5.0	- 1.6
527+98 to 529+06	110.0	113.0	- 3.0	- 2.7
529+80 to 531+25	143.0	143.0	0.0	0.0
Totals	2463.0	2543.0	-80.0	- 3.1

measured during the plotting operation. The two sections obscured by snow were later identified by stereoscopic examination of the air-photos. In addition to the two sections which were completely undetected, the exact delineation and measurement of other sections was questionable because of the presence of snow, silt, and debris near the ends of the sections. On sections which were positively identified and delineated, the magnitude of the errors was much smaller. Three of these sections were in exact agreement with the field measurements of the construction record.

Most of the errors were relatively small and in the minus direction. One large plus error occurred on the section between stations 456+97 and 463+88. It is believed that the greater portion of this error is attributable to a mistake in the identification of the ends of the section.

The total length of paved side ditch by field measurements was 2,543 feet compared to 2,463 feet by photogrammetric measurements. The error of -80 feet, or -3.1 percent, is misleading since it results largely from the two sections which were completely omitted in the photogrammetric measurements. If these two sections were not considered in the total, the error would be only 16 feet, or 0.7 percent. Even this error is deceptive since it results from the compensation of one large plus error and several small minus errors. Perhaps a better indication of measurement accuracy is obtained by also omitting the section with the large plus error. The error for the total would then be 27.5 feet, or 1.5 percent.

It is believed that the smaller errors, or measurement errors, are a better indication of the accuracy of photogrammetric measurements. Even so, errors in identification must be taken into account as they can be expected to occur in the general application of photogrammetric methods. In light of the large errors in identification, it is doubtful that photogrammetric measurement of paved side ditch is feasible. Even if errors in identification were not a problem, the ease and certainty with which these measurements can be made in the field makes photogrammetric measurement questionable.

Sodding

Photogrammetric measurements of sodding were compared on the basis of areas between 100 foot stations along the highway. For these comparisons, it was necessary to re-group the construction record quantities to correspond with the 100 foot lengths used in computing the photogrammetric quantities. Irregular-shaped areas were planimetered while areas of more constant dimensions were subdivided into simple geometric shapes from which the quantities were computed from the scaled dimensions. Slope corrections were applied in both the longitudinal and transverse directions. In all, 94 sections of sodding were measured and compared. Table 13

TABLE 13

Sodding Measurements

Station	Photogrammetric Measurement Sq. Ft.	Field Measurement Sq. Ft.	Error Sq. Ft.	% Error
445				
446	905	898	7	0.8
447	610	642	- 32	- 5.0
448	715	749	- 34	- 4.5
449	910	1310	- 400	- 30.5
450	1275	958	317	33.1
451	980	1806	- 826	- 45.7
452	830	1725	- 895	- 51.8
453	665	869	- 204	- 23.5
454	870	838	30	3.6
455	440	564	- 124	- 22.0
456	440	560	- 120	- 21.4
457	530	560	- 30	- 5.4
458	700	560	140	25.0
459	700	560	140	25.0
460	700	560	140	25.0
461	700	560	140	25.0
462	705	560	145	25.9
463	725	560	165	29.5
464	905	953	- 48	- 5.0
465	2315	2548	- 233	- 9.1
466	2020	2180	- 160	- 7.3
467	1905	2245	- 340	- 15.1
468	1990	2140	- 150	- 7.0
469	1810	2140	- 330	- 15.4
470	1765	1623	142	8.8
471	2100	2495	- 395	- 15.8
472	1650	2296	- 646	- 28.2
473	1700	2097	- 397	- 18.9
474	1785	1863	- 78	- 4.2
475	2380	2733	- 353	- 12.9
476	1550	1800	- 250	- 13.9
477	1715	1825	- 110	- 6.0
478	1790	1850	- 60	- 3.2
479	1840	1825	15	0.8
480	1650	1748	- 98	- 5.6

TABLE 13 (Continued)

Station	Photogrammetric Measurement Sq. Ft.	Field Measurement Sq. Ft.	Error Sq. Ft.	% Error
481	1885	1907	- 22	- 1.2
482	1735	1727	8	0.5
483	1400	1094	306	27.9
484	515	549	- 34	- 6.2
485	555	560	- 5	- 0.9
486	775	770	5	0.6
487	1230	1355	- 125	- 9.2
488	1245	1292	- 47	- 3.6
489	895	1312	- 417	- 31.8
490	555	560	- 5	- 0.9
491	600	560	40	7.1
492	560	560	0	0.0
493	1045	1169	- 124	- 10.6
494	1585	1767	- 182	- 10.3
495	830	1911	-1081	- 56.6
496	1265	1754	- 489	- 27.9
497	1620	1783	- 163	- 9.1
498	1780	2915	-1135	- 38.8
499	1430	1638	- 208	- 12.7
500	1275	1872	- 597	- 31.9
501	440	840	- 400	- 47.7
502	535	840	- 305	- 36.3
503	505	840	- 335	- 39.9
504	485	884	- 399	- 45.2
505	1320	1425	- 105	- 7.4
506	1150	1180	- 30	- 2.5
507	1575	1638	- 63	- 3.8
508	1615	1718	- 103	- 6.0
509	1600	1731	- 131	- 7.6
510	1615	1744	- 129	- 7.4
511	1660	1757	- 97	- 5.5
512	1680	1770	- 90	- 5.1
513	1540	1783	- 243	- 13.6
514	1260	1496	- 236	- 15.8
515	660	1044	- 384	- 36.8
516	1070	1701	- 631	- 37.1
517	1470	1703	- 233	- 13.7
518	990	1007	- 17	- 1.7
519	635	635	0	0.0
520	555	560	- 5	- 0.9

TABLE 13 (Continued)

Station	Photogrammetric Measurement Sq. Ft.	Field Measurement Sq. Ft.	Error Sq. Ft.	% Error
521	555	1403	- 848	- 60.4
522	520	710	- 190	- 26.8
523	2390	2425	- 35	- 1.4
524	2370	2466	- 96	- 3.9
525	1045	1902	- 857	- 45.0
526	1020	1947	- 927	- 47.6
527	565	1311	- 746	- 56.9
528	555	1311	- 756	- 57.6
529	740	1036	- 296	- 28.6
530	635	780	- 145	- 18.6
531	650	511	139	27.2
532	1620	1856	- 236	- 12.7
533	1320	1986	- 666	- 33.5
534	1515	1904	- 389	- 20.4
535	1500	1822	- 322	- 17.7
536	1610	1739	- 129	- 7.4
537	1525	1621	- 96	- 5.9
538	675	944	- 269	- 28.5
538+55	265	317	- 52	- 16.4
Total	110,485	130,542	-20,057	- 15.4

shows these comparisons.

Errors in identification were made almost consistently in the sodding measurements. The lack of contrast in tone between sodded areas and seeded areas made identification difficult, if not impossible, in many places. It was impossible to distinguish between sodding which existed prior to construction and newly placed sodding. The dark tones present in wet spots were sometimes mistaken for sodded areas. Snow cover added to the difficulty in identification in the ditches and on some side slopes. A sharper contrast in tones, such as exists during the spring of the year, might have improved the identifying characteristics of the sodding considerably.

Even though gross identification errors were numerous, errors in measurement probably accounted for a significant portion of the total error for each section. A difference of a few inches in a narrow band of sodding caused a large percentage error. This type of error was unavoidable since measurements could be made only to the nearest 0.5 of a foot.

Of the 94 sections compared, 76 were in error in the minus direction. Most large errors were in the minus direction; plus errors were generally small. Where the identification of sodding was in doubt, the area in question was not measured and recorded as sodding. This probably accounts for the predominance of minus errors.

The quantity errors ranged from zero to -1135 square feet. The percentage errors ranged from zero to -60.4 percent. The total area by field measurement was 130,542 square feet compared to 110,485 square feet by photogrammetric measurement for an error of -20,057 square feet, or -15.4 percent. An error of this magnitude would represent a considerable loss to the contractor for work done.

The results of these comparisons would seem to indicate that photogrammetric measurement of sodding under the conditions of this study is not accurate enough for final pay purposes.

Curbing

Poor agreement was obtained between the field and photogrammetric measurements of curbing. A total of 1607.0 feet of integral curbing was measured photogrammetrically compared to 1227.2 feet by field measurement for an error of 379.8 feet, or 30.9 percent. Table 14 shows the comparisons of quantities between 100 foot stations along the highway. For these comparisons, the construction record quantities were re-grouped to correspond with the 100 foot lengths used in grouping the photogrammetric quantities. Nine individual sections were compared.

The quantity errors ranged from zero to 116.6 feet, and the percentage errors ranged from zero to 178.2 percent. Most of the errors were large and all but one were in the plus direction.

TABLE 14

Curbing Measurements

Station	Photogrammetric Measurement (Feet)	Field Measurement (Feet)	Error (Feet)	% Error
445				
446	201.0	175.7	25.3	14.4
447	178.5	133.8	44.7	33.4
448	202.5	169.0	33.5	19.8
449	182.0	65.4	116.6	178.2
450	195.0	169.4	25.6	15.1
451	212.5	150.2	53.3	33.5
452	198.0	116.2	81.8	70.4
453	200.0	200.0	0.0	0.0
454	37.5	38.5	- 1.0	- 2.6
Total	1607.0	1227.2	379.8	30.9

Again, errors in identification were responsible for a large part of the discrepancy between the photogrammetric and the field measurements. Lengths of curbing which had existed prior to construction were unknowingly included with the new curbing in the photogrammetric measurements. Although there was no combined curb and gutter construction on the portion of the project chosen for study, observations several feet in advance of the beginning point for the study indicated that a definite distinction between the two types would be difficult, if not impossible.

Even though there was little difficulty in delineating and measuring old and new curbing together, the large errors which resulted from the inability to distinguish between the old and new make photogrammetric measurement of curbing impractical.

Guard Rail

Relatively good agreement between the field and photogrammetric measurements was obtained in the guard rail comparisons. The total length of guard rail by field measurement was 5542.5 feet compared to 5531.5 feet by photogrammetric measurement for an error of -11.0 feet, or -0.2 percent. The measurement errors ranged from zero to -0.7 percent. The errors for the individual sections were small, and all but one were in the minus direction.

The lengths of guard rail were grouped and compared in accordance with the quantity subdivisions of the construction

TABLE 15

Guard Rail Measurements

Approx. Sta. Interval	Photo. Measurement (Feet)	Field Measurement (Feet)	Error (Feet)	% Error
453+50 to 462+62 Rt	930.0	930.0	0	0
498+78 to 503+74 Rt	491.0	492.5	- 1.5	- .3
517+34 to 528+54 Rt	1103.0	1105.0	- 2.0	- .2
513+73 to 514+36 Rt	67.5	67.5	0	0
528+58 to 529+83 Rt	105.5	105.0	0.5	.5
452+76 to 462+84 Lt	979.0	980.0	- 1.0	- .1
499+40 to 505+48 Lt	591.5	592.5	- 1.0	- .2
521+65 to 525+85 Lt	428.5	430.0	- 2.0	- .5
2+43 Pvt Drive to 529+33 Lt	377.5	380.0	- 2.5	- .7
518+06 to 521+30 Lt	328.0	330.0	- 2.0	- .6
514+22 to 515+47 Lt	130.0	130.0	0	0
Total	5531.5	5542.5	-11.0	- .2

record. Because of the small grades, no slope corrections were applied to the photogrammetric measurements. The comparisons are shown in Table 15.

Errors in identification were no problem in determining guard rail lengths. The features of the guard rail were sharply defined, and delineation and measurement were accomplished with considerable ease and certainty.

The results of these comparisons would seem to indicate that fairly accurate measurements of guard rail can be obtained photogrammetrically.

Guide Post

Guide posts were outlined on the photogrammetric manuscript and counted in groups. These groups, designated by an approximate station interval along the highway, corresponded to those listed by the construction record. In all, 12 groups were counted and compared. These comparisons are shown in Table 16.

The field count listed a total of 46 guide posts compared to the photogrammetric count of 54. Three of the 12 groups were in error from the construction record count.

Although no measurement errors were involved in the guide post counts, errors in identification were possible and did occur. One guide post was simply overlooked in the photogrammetric count of the group at station 528+75. Eight guide posts were counted in the station interval from 529+00 to 529+80 in which no guide posts were listed by the construction

TABLE 16

Guide Post Count

Approx. Sta. Interval	Photo. Count	Field Count	Error
451+64 to 453+20	5	5	0
463+00 to 464+15	1	1	0
482+00 "F" to 482+00 "A"	9	9	0
489+40 to 493+95	1	1	0
494+50 to 498+80	1	1	0
503+75 to 513+80	2	1	1
514+95 to 517+45	1	1	0
524+00 to 525+50	4	4	0
528+75 (Pvt. Drive	5	6	-1
529+00 to 529+80	8	0	8
530+00 to 530+90	3	3	0
Line S-11-A	14	14	0
Total	54	46	8

record. It is believed that these guide posts were extant before construction and were, therefore, not included in the final pay quantities of the construction record. The photogrammetric count showed two guide posts in the station interval from 503+75 to 513+80; whereas, the construction record listed only one. It may be that the construction record was in error on this count since these posts were located on newly constructed fill and photogrammetric identification was fairly certain.

The inability to distinguish between new guide posts and those placed prior to construction would seem to indicate that photogrammetric counting is unreliable. However, it should be recognized that most of these errors would not occur on projects in which all construction is new.

EVALUATION OF STUDY CONDITIONS

In reviewing the conditions under which this study was performed, it would appear that there were inadequacies and limitations, some of which were unforseeable and some of which were unavoidable, which may have interfered with the intended purposes of the study. Among these is the inability to state definitely and emphatically whether photogrammetric measurements are accurate enough for use in making final payment. This inability arises from the previously mentioned fact that there are no commonly accepted tolerance limits for the accuracy of final pay measurements. From this study, it is only possible to present comparisons between the photogrammetric and conventional methods of determination. Conclusions regarding the acceptance or rejection of photogrammetric final pay quantities must be limited to commentary on the degree of agreement between the comparisons. It would appear that the suitability of photogrammetric final pay quantities is a matter which can be decided only by the contractors who perform the work and must accept payment on the basis of these quantities.

Confusion often occurred in distinguishing between items which existed prior to construction and those which were to be measured for final payment as new construction. This confusion might have been avoided had a section of newly located highway been selected for study rather than a relocation of an already existing highway. Such a selection would have resulted in fewer identification errors with a subsequently better indication of measurement accuracy. However, the difficulties in identification which were encountered must be anticipated in any general application of photogrammetric methods to final pay quantity measurement. Any attempt to control this aspect of the study would, therefore, be of doubtful value.

Another limiting factor was the snow cover which obscured portions of the highway and made plotting and measurement difficult. Even though the amount of snow was generally very small, it was present in sufficient amounts at critical places to cause trouble. This was particularly true of the paved side ditches. Had this been anticipated beforehand, the photography could have been flown at a time when the ground was completely free of snow. The most opportune time would have been immediately after the completion of construction. This would also have eliminated the further complication of weathering and erosion.

Inadequate cross section terrain data obtained by field surveys greatly limited the value of comparisons between photogrammetric and construction record quantities and prevented a complete evaluation of earthwork quantities in as much as the quantities for the side road entrances could not be determined. Full cross-sectional areas for all cross sections could have been computed if the terrain data had been extended far enough from the centerline to cover all earthwork operations. It would have been possible to extend the terrain cross sections to meet the slope stake elevation readings of the template cross sections, but this involves an element of uncertainty. Apparently this was done by the highway department in computing the construction record quantities for those cross sections which were deficient in terrain data. In any event, reliable comparisons were obtained with the electronically computed field quantities since the cross-sectional areas were computed by using the same shortened terrain data as were used in computing the photogrammetric cross-sectional areas. Additional terrain cross sections were apparently taken in the field for the side road entrances prior to construction, but the data were not recorded in the field cross section books and, consequently, no earthwork quantities could be developed. As pointed out previously, it would have been possible, and in many respects desirable, to have obtained the terrain cross sections by photogrammetric methods. This would have allowed accurate extension of the terrain cross sections even after the completion of construction. It also would have eliminated some of the confusion as to which quantities were to be measured for final payment and which existed prior to construction.

Finally, the important factors of time and cost were not considered because of the necessarily limited scope of the study and because the conditions under which the study was performed could hardly be considered typical of those which would prevail in actual practice. Due to the inexperience of the author with photogrammetric plotting equipment, a greater amount of time was consumed in the orientation and plotting of the models than should be anticipated in the routine of actual practice. A rough log of the time showed that 224 hours were consumed in the orientation, plotting, and measurement of the models. Since the models averaged about 1000 feet in length, about 2 1/2 hours were required for each 100 feet of length. Unfortunately, a breakdown of the time required for each type of pay quantity was not possible since all quantities were plotted and measured concurrently.

SUMMARY AND CONCLUSIONS

In conducting this study, an attempt was made to approximate typical conditions and situations. It must be recognized, however, that no section of highway can be classified as typical. Variations in terrain, land use, and physical features of highways make this an impossibility. Nor can the equipment or procedures which were employed be classified as representative. A wide variety of photogrammetric plotting instruments, flying heights, and plotting procedures could be used in an undertaking of this nature. Caution must be exercised, therefore, in any generalization of the results and conclusions of this particular study.

From the experience gained in the execution of this study, it may be concluded that the photogrammetric methods and procedures described herein are applicable, with limited regard for accuracy, to the measurement of the following final pay quantities: earthwork, concrete pavement and appurtenances, paved side ditch, sodding, curbing, guard rail, and guide posts.

From the analysis and results of the accuracy comparisons of this study, it may be concluded that:

1. There was good agreement between the photogrammetric earthwork excavation quantities and the corresponding quantities computed electronically from the field cross section books.

(The photogrammetric excavation quantities were in error by -3.5 percent before adjustment and by -2.0 percent after adjustment.) The photogrammetric borrow pit quantities showed similarly good agreement with the field quantities. (Borrow Pit 3 was in error by -2.0 percent before adjustment and by 0.8 percent after adjustment. Borrow Pit 4 was in error by -0.5 percent both before and after adjustment.)

2. There was generally poor agreement between the photogrammetric earthwork excavation quantities and the corresponding construction record quantities because of inadequate original terrain data. (The adjusted and unadjusted photogrammetric quantities were in error by -9.9 and -11.3 percent. For Borrow Pit 3, the adjusted and unadjusted photogrammetric quantities were in error by 3.2 and 0.4 percent, and for Borrow Pit 4, the adjusted and unadjusted photogrammetric quantities were in error by 4.7 and 4.6 percent.)
3. The photogrammetric embankment quantities showed good agreement with the corresponding quantities computed electronically from the field cross section books. (The adjusted and

unadjusted quantities were in error by -2.0 and -2.5 percent.)

4. Adjustment of the photogrammetric earthwork quantities to an accurate centerline profile generally improved accuracy.
5. The relatively large errors of some of the individual sections of earthwork compensated to yield a smaller error for the total earthwork.
6. The percentage errors generally varied inversely with the depth and size of the earthwork quantity.
7. The photogrammetric measurements of concrete pavement and appurtenances were in close agreement with the construction record measurements. (The percentage error was, for all practical purposes, zero.)
8. Due to the compensation of a few large errors, the photogrammetric measurements of paved side ditch showed fairly good agreement with the construction record. (The photogrammetric measurements were in error by -3.1 percent.)
9. Poor agreement was obtained in the comparison of photogrammetric quantities of sodding with the corresponding construction record quantities. (The error was -15.4 percent.)
10. Poor agreement was obtained in the photogrammetric

and construction record comparisons of curbing. (The error was 30.9 percent.)

11. The photogrammetric measurements of guard rail were in close agreement with the construction record measurements. (The photogrammetric quantities were in error by -0.2 percent.)
12. The photogrammetric count of guide posts did not agree well with the construction record count. (The construction record listed a total of 46 guide posts compated to the photogrammetric count of 54.)

From the analysis and results of the vertical accuracy study, it may be concluded that:

1. The distribution of centerline elevation errors closely followed a normal distribution curve. Compensation of errors should be expected with this type of distribution.
2. The calculated C-factor was approximately 1070, a comparitively high value.
3. The mean of all centerline elevation errors was not significantly different from zero.
4. Systematic errors which varied from model to model were operative.
5. The precision of elevation reading is dependent not only upon the model but also upon the point at which the reading is made within the model.

As a corollary to the above conclusions, it may be stated that the photogrammetric techniques described herein definitely provide accuracy and reliability of a nature that would warrant their use in the location and design phases of highway construction. This is true not only for the preliminary stages but also for the final stages of location and design as well.

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APPENDIX

TABLE I

EARTHWORK QUANTITY COMPARISONS
(Cu Yd)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Station	Constr. Record Cut	Field Cut	Unadj. Photo. Cut	Adj. Photo. Cut	% Error (4-2)/2	% Error (5-2)/2	% Error (4-3)/3	% Error (5-3)/3	Field Fill	Unadj. Photo. Fill	Adj. Photo Fill	% Error (11-10)/10	% Error (12-10)/10	Net Volume Field	Net Volume Unadj. Data	Net Volume Adj. Data
44500	Line "F"															
44600	367	158	253	188	- 31.1	- 48.8	60.1	19.0	6	1	8	- 83.3	33.3	152	252	180
44700	284	94	104	108	- 29.4	- 62.0	117.0	14.9	5	6	29	0.0	383.3	88	198	79
44800	317	86	138	100	- 56.5	- 68.5	60.5	16.3	8	9	26	12.5	225.0	78	129	74
44838	127	50	57	59	- 55.1	- 53.5	14.0	12.5	3	1	1	- 66.7	- 66.7	47	56	58
44900	203	81	85	92	- 58.1	- 54.7	4.9	13.6	5	7	6	40.0	20.0	76	78	86
45000	333	147	104	175	- 68.8	- 48.5	- 29.3	19.0	10	42	11	320.0	10.0	137	64	160
45100	359	167	105	194	- 70.8	- 46.0	- 37.1	16.2	22	50	12	127.3	- 45.5	145	55	182
45200	185	62	44	77	- 76.2	- 58.4	- 29.0	24.2	526	563	508	7.0	- 3.4	- 490	- 519	- 443
45250	17	3	0	1	-100.0	- 94.1	-100.0	33.3	464	500	495	7.4	3.3	- 463	- 500	- 494
45280	7	19	11	19	36.4	172.4	- 38.5	1.5	230	246	242	17.0	4.8	- 252	- 283	- 243
45300	0	15	8	17	100.0	——	- 46.7	13.3	218	249	214	14.2	- 1.8	- 203	- 241	- 197
45335	12	19	10	24	- 16.7	100.0	- 47.4	26.3	662	703	640	6.2	- 3.3	- 643	- 693	- 616
45365	17	18	13	22	- 23.5	29.4	- 27.8	22.2	834	848	797	1.6	- 4.7	- 816	- 835	- 775
45420	14	15	11	14	- 21.4	0.0	- 26.7	- 6.7	1160	1160	1138	0.0	- 1.9	-1145	-1149	-1124
45440	20	22	16	28	- 20.0	40.0	- 27.3	27.3	1414	1413	1374	- 0.2	- 3.0	-1394	-1397	-1346
45485	30	17	15	30	- 50.0	- 6.3	- 11.8	76.5	1818	1823	1758	0.2	- 3.3	-1801	-1808	-1728
45500	10	10	16	16	60.0	60.0	60.0	60.0	730	730	724	0.0	- 0.8	- 720	- 714	- 708
45575	19	28	48	52	152.6	173.7	71.4	85.7	1838	1799	1757	- 2.1	- 4.4	-1810	-1751	-1705
45600	15	50	96	99	540.0	560.0	92.0	98.0	9715	9313	9332	- 4.2	- 3.9	-9665	-9217	-9233
45615	0	2	4	4	——	——	100.0	100.0	2358	2283	2299	- 2.9	- 2.3	-2356	-2279	-2295
45628	0	0	0	0	0.0	0.0	0.0	0.0	2354	2283	2295	- 3.0	- 2.5	-2354	-2283	-2295
45645	0	0	0	0	0.0	0.0	0.0	0.0	3102	3021	3042	- 2.6	- 1.9	-3102	-3021	-3042
45680	56	56	70	70	25.0	25.0	25.0	25.0	5971	6090	6119	2.0	2.5	-5915	-6020	-6049
45700	45	45	61	59	35.6	31.1	35.6	31.1	3284	3361	3377	2.3	2.8	-3238	-3300	-3318
45750	34	38	56	50	64.7	47.1	47.4	31.6	7884	7906	8011	0.3	1.6	-7846	-7850	-7961
45800	6	9	7	5	16.7	- 16.7	- 22.2	- 44.4	7335	7513	7578	2.4	3.3	-7326	-7506	-7573
45850	4	7	3	3	- 25.0	- 25.0	- 57.1	- 57.1	6938	7089	7133	2.2	2.8	-6931	-7086	-7130
45900	3	4	1	0	- 66.7	-100.0	- 75.0	-100.0	6194	6305	6388	1.8	3.1	-6190	-6304	-6388
45950	0	4	1	0	——	0.0	- 75.0	-100.0	5583	5709	5749	2.3	3.0	-5579	-5708	-5749
46000	0	0	0	0	0.0	0.0	0.0	0.0	5300	5400	5400	1.9	1.9	-5300	-5400	-5400
46060	0	0	3	0	——	0.0	——	0.0	5948	6009	6004	1.0	0.9	-5948	-6006	-6004
46100	0	0	2	0	——	0.0	——	0.0	3603	3651	3634	1.3	0.9	-3603	-3649	-3634
46150	0	1	0	0	0.0	0.0	100.0	-100.0	3817	3825	3808	0.2	0.2	-3816	-3825	-3808
46200	0	1	4	1	——	——	300.0	0.0	2779	2712	2743	- 2.4	- 1.3	-2778	-2708	-2742
46240	0	3	10	5	——	——	233.3	66.7	1277	1248	1288	- 2.3	- 0.9	-1274	-1238	-1283
46280	7	17	13	11	85.7	57.1	- 23.5	- 35.3	595	593	608	- 0.3	2.2	- 578	- 580	- 597

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TABLE I (Continued)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Station	Cons't. Record	Field Cut	Unadj. Photo. Cut	Adj. Photo. Cut	% Error $\frac{4-2}{2}$	% Error $\frac{5-2}{2}$	% Error $\frac{4-3}{3}$	% Error $\frac{5-3}{3}$	Field Fill	Unadj. Photo. Fill	Adj. Photo Fill	% Error $\frac{11-10}{10}$	% Error $\frac{12-10}{10}$	Net Volume Field	Net Volume Unadj. Data	Net Volume Adj. Data
46300	10	17	10	10	0.0	0.0	- 41.2	- 41.2	171	172	172	0.6	0.6	- 161	- 162	- 162
46355	85	90	82	71	- 3.5	- 16.5	- 8.9	- 21.1	232	240	238	- 3.4	2.6	- 142	- 164	- 167
46400	194	170	166	145	- 15.5	- 25.3	- 1.2	- 14.7	46	36	53	- 21.7	15.2	124	130	92
46470	790	670	695	629	- 12.5	- 17.2	- 3.7	- 6.1	12	7	17	- 41.7	41.7	658	628	612
46500	556	505	512	490	- 7.9	- 18.7	1.4	- 3.0	0	0	0	0.0	0.0	505	512	490
46550	1270	1150	1125	1081	- 11.4	- 14.9	- 2.2	- 6.0	0	0	0	0.0	0.0	1150	1125	1081
46600	1533	1429	1350	1323	- 13.3	- 16.9	- 5.5	- 7.4	0	0	0	0.0	0.0	1429	1350	1323
46655	2017	1825	1775	1742	- 12.0	- 13.6	- 2.7	- 4.5	0	0	0	0.0	0.0	1825	1775	1742
46700	1888	1759	1750	1711	- 7.3	- 9.4	- 0.5	- 2.7	0	0	0	0.0	0.0	1759	1750	1711
46775	3425	3249	3233	3218	- 5.6	- 6.0	- 1.1	- 1.6	0	0	0	0.0	0.0	3249	3233	3218
46800	1567	1489	1441	1488	- 8.0	- 5.0	- 3.2	- 0.1	0	0	0	0.0	0.0	1489	1441	1488
46845	2058	1974	1873	1933	- 9.0	- 6.1	- 5.1	- 2.1	0	0	0	0.0	0.0	1974	1873	1933
46885	1288	1268	1205	1219	- 6.4	- 5.4	- 5.0	- 3.9	4	12	9	200.0	125.0	1264	1193	1210
46900	325	320	314	302	- 3.1	- 1.2	- 4.3	- 2.1	7	13	11	85.7	57.1	313	301	291
46960	533	564	530	574	- 0.6	7.7	- 5.7	1.8	51	67	44	31.4	- 13.7	513	463	530
46975	167	194	167	200	0.0	19.9	- 13.9	3.1	36	43	51	38.3	- 11.7	134	86	153
47000	76	84	59	81	- 22.4	20.5	- 29.8	0.0	49	79	56	61.2	6.1	35	- 20	25
47050	103	79	33	90	- 68.0	- 12.6	- 58.2	13.9	37	87	34	135.1	- 8.1	42	- 54	56
47100	148	128	52	99	- 64.9	- 33.1	- 51.9	- 8.3	25	51	41	204.0	- 26.7	103	1	58
47155	281	237	164	209	- 41.6	- 28.2	- 30.8	- 11.8	14	33	37	135.7	21.4	223	131	172
47200	392	336	267	315	- 31.9	- 19.6	- 20.5	- 6.3	11	30	28	172.7	154.5	325	237	287
47225	426	379	335	376	- 21.4	- 11.7	- 11.9	- 0.8	4	23	7	475.0	75.0	375	312	369
47300	1047	985	886	937	- 15.4	- 10.5	- 10.1	- 4.9	4	12	6	200.0	50.0	981	874	931
47350	986	947	849	901	- 13.9	- 8.6	- 10.3	- 4.9	1	5	2	400.0	100.0	946	844	899
47400	1159	1184	1099	1142	- 5.2	- 1.5	- 7.2	- 3.5	0	2	0	—	0.0	1184	1097	1142
47500	2733	2742	2640	2689	- 3.4	- 1.6	- 3.7	- 1.9	0	3	0	—	0.0	2742	2637	2689
47550	1567	1344	1388	1431	- 10.3	- 8.8	- 10.1	- 8.6	0	4	2	—	—	1344	1384	1429
47600	1643	1667	1517	1498	- 8.0	- 8.8	- 9.3	- 10.1	12	13	15	8.3	25.0	1655	1499	1483
47700	3273	3199	3157	3181	- 3.5	- 2.5	- 3.1	- 2.1	24	22	26	- 8.3	8.3	3175	3135	3155
47750	1582	1592	1538	1590	- 2.8	- 0.5	- 3.4	- 2.6	0	0	0	0.0	0.0	1592	1538	1590
47800	1502	1520	1461	1433	- 3.9	- 4.6	- 3.9	- 5.7	0	0	0	0.0	0.0	1520	1461	1433
47850	1320	1334	1276	1270	- 3.3	- 3.8	- 4.3	- 4.8	0	0	0	0.0	0.0	1334	1276	1270
47900	1031	1036	965	996	- 6.4	- 3.4	- 6.9	- 3.9	2	3	3	50.0	50.0	1034	962	993
47950	709	695	651	664	- 8.2	- 6.3	- 6.3	- 4.5	3	4	4	33.3	33.3	692	647	660
48000	436	414	426	391	- 2.3	- 10.3	2.9	- 5.6	8	3	7	- 62.5	- 12.5	406	423	384
48025	161	161	176	149	9.3	- 7.5	24.8	3.7	6	2	4	- 66.7	- 33.3	155	174	145
48045	254	214	279	234	9.3	- 7.9	28.5	9.3	11	18	19	63.6	72.7	203	257	215
48100	220	187	221	184	0.5	- 13.9	18.2	- 1.6	6	15	17	150.0	183.3	181	206	167
48165	429	371	419	351	- 2.3	- 18.2	12.9	- 5.4	0	0	4	0.0	—	371	419	347
48200	213	181	201	171	- 5.6	- 19.7	10.5	- 5.5	10	3	6	- 70.0	- 40.0	171	198	165
48250	195	169	188	160	- 3.6	- 17.9	11.2	- 5.3	117	105	117	- 10.3	0.0	52	83	43
48285	50	51	53	45	6.0	- 10.0	3.9	- 11.8	235	229	241	- 2.6	2.6	- 184	- 176	- 196
48300	6	7	6	5	0.0	- 16.7	- 14.3	- 28.6	155	150	154	- 3.2	- 0.6	- 148	- 144	- 149

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TABLE I (Continued)

1 Station	2 Cons't. Record Cut	3 Field Cut	4 Unadj. Photo. Cut	5 Adj. Photo. Cut	6 % Error $\frac{4-2}{2}$	7 % Error $\frac{5-2}{2}$	8 % Error $\frac{4-3}{3}$	9 % Error $\frac{5-3}{3}$	10 Field Fill	11 Unadj. Photo. Fill	12 Adj. Photo Fill	13 % Error $\frac{11-10}{10}$	14 % Error $\frac{12-10}{10}$	15 Net Volume Field	16 Net Volume Unadj. Data	17 Net Volume Adj. Data
48303	2	2	2	1	0.0	- 50.0	0.0	- 50.0	35	33	34	- 5.7	- 2.9	- 33	- 31	- 33
48015	Line "A"															
48055	15	13	12	10	- 20.0	- 33.3	- 7.7	- 23.1	637	602	631	- 5.5	- 0.9	- 624	- 590	- 621
48100	3	3	1	0	- 66.7	-100.0	- 66.7	-100.0	727	692	723	- 4.8	- 0.6	- 724	- 691	- 723
48144	5	4	0	0	-100.0	-100.0	-100.0	-100.0	1098	1048	1058	- 4.6	- 3.6	-1094	-1048	-1058
48200	0	1	2	1	----	----	100.0	0.0	836	802	808	- 4.1	- 3.3	- 835	- 800	- 807
48230	3	4	2	2	- 33.3	- 33.3	- 50.0	- 50.0	569	560	560	- 1.6	- 1.6	- 565	- 558	- 558
48262	4	5	1	2	- 75.0	- 50.0	- 80.0	- 60.0	511	500	487	- 2.2	- 4.7	- 506	- 499	- 485
48285	1	3	1	2	0.0	100.0	- 66.7	- 33.3	242	236	225	- 2.5	- 7.0	- 239	- 235	- 223
48300	4	5	3	4	- 25.0	0.0	- 40.0	- 20.0	110	110	104	0.0	- 5.5	- 105	- 107	- 100
48345	68	80	72	67	5.9	- 1.5	- 10.0	- 16.3	159	163	164	2.5	3.1	- 79	- 91	- 97
48400	315	322	291	269	- 7.6	- 14.6	- 9.6	- 16.5	41	26	36	- 36.6	- 12.2	281	265	233
48500	1063	1015	917	893	- 13.7	- 16.0	- 9.7	- 12.0	2	2	4	0.0	100.0	1013	915	889
48600	1002	904	806	846	- 19.6	- 15.6	- 10.9	- 6.4	4	6	4	50.0	0.0	900	800	842
48700	432	367	328	372	- 24.1	- 13.9	- 10.6	1.4	109	102	91	- 6.4	- 16.5	258	226	281
48800	61	60	68	70	11.5	14.8	13.3	16.7	450	434	441	- 3.6	- 2.0	- 390	- 366	- 371
48865	40	36	52	50	30.0	25.0	44.4	38.9	434	422	433	- 2.8	- 0.2	- 398	- 370	- 383
48900	16	14	23	23	43.8	43.8	64.3	64.3	231	216	216	- 6.5	- 6.5	- 217	- 193	- 193
48933	2	3	5	5	150.0	150.0	66.7	66.7	171	155	155	- 9.4	- 9.4	- 168	- 150	- 150
49000	135	109	121	102	- 10.4	- 24.4	11.0	- 6.4	128	112	114	- 12.5	- 10.9	- 19	9	- 12
49100	1109	1030	1024	996	- 7.7	- 10.2	- 0.6	- 3.3	9	3	6	- 66.7	- 33.3	1021	1021	990
49130	581	548	556	551	- 4.3	- 5.2	1.5	0.5	0	0	0	0.0	0.0	548	556	551
49200	1290	1395	1444	1423	11.9	10.3	3.5	2.0	0	0	0	0.0	0.0	1395	1444	1423
49300	1193	1365	1430	1402	19.9	17.5	4.8	2.7	0	0	0	0.0	0.0	1365	1430	1402
49380	336	292	362	315	7.7	- 6.3	24.0	7.9	76	33	74	- 56.6	- 2.6	216	329	241
49400	14	15	27	14	92.9	0.0	80.0	- 6.7	47	27	47	- 42.6	0.0	- 32	0	- 33
49430	15	19	27	14	80.0	- 6.7	42.1	- 26.3	86	60	86	- 30.2	0.0	- 67	- 33	- 72
49500	76	59	85	58	11.8	- 23.7	44.1	- 1.7	125	85	124	- 32.0	- 0.8	- 66	0	- 66
49600	498	425	480	408	- 3.6	- 18.1	12.9	- 4.0	30	16	34	- 46.7	13.3	395	464	374
49635	327	297	331	293	1.2	- 10.4	11.4	- 1.3	1	0	0	-100.0	-100.0	296	331	293
49700	692	641	723	661	4.5	- 4.5	12.8	3.1	2	1	2	- 50.0	0.0	639	722	659
49750	525	481	541	494	3.0	- 5.9	12.5	2.7	3	1	5	- 66.7	66.7	478	540	489
49800	466	406	442	415	- 5.2	- 10.9	8.9	2.2	2	2	5	0.0	150.0	404	440	410
49845	296	246	261	253	- 11.8	- 14.5	6.1	2.8	11	8	11	- 27.3	0.0	235	253	242
49870	103	94	108	103	4.9	0.0	14.9	9.6	47	43	44	- 8.5	- 6.4	47	65	59
49900	60	72	105	100	75.0	66.7	45.8	38.9	290	276	280	- 4.8	- 3.4	- 218	- 171	- 180
49918	7	17	37	34	428.6	385.7	117.6	100.0	388	362	364	- 6.7	- 6.2	- 371	- 325	- 330
49934	4	13	18	19	350.0	375.0	38.5	46.2	541	508	504	- 6.1	- 6.8	- 528	- 490	- 485
49963	11	37	69	76	527.3	590.9	86.5	105.4	1615	1526	1509	- 5.5	- 6.6	-1578	-1457	-1433
49973	8	13	25	26	212.5	225.0	92.3	100.0	770	736	737	- 4.4	- 4.3	- 757	- 711	- 711

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TABLE I (Continued)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Station	Const. Record Cut	Field Cut	Unadj. Photo. Cut	Adj. Photo. Cut	% Error $\frac{4-2}{2}$	% Error $\frac{5-2}{2}$	% Error $\frac{4-3}{3}$	% Error $\frac{5-3}{3}$	Field Fill	Unadj. Photo. Fill	Adj. Photo Fill	% Error $\frac{11-10}{10}$	% Error $\frac{12-10}{10}$	Net Volume Field	Net Volume Unadj. Data	Net Volume Adj. Data
49990	8	9	20	22	150.0	175.0	122.2	144.4	1615	1594	1588	- 1.3	- 1.7	-1606	-1574	-1566
50000	0	7	20	25	---	---	185.7	257.1	1005	1002	989	- 0.3	- 1.6	- 998	- 982	- 964
50050	4	31	66	81	---	---	112.9	161.3	4795	4713	4668	- 1.7	- 2.7	-4765	-4647	-4587
50100	118	0	0	0	100.0	-100.0	0.0	0.0	4541	4429	4384	- 2.5	- 3.5	-4541	-4429	-4384
50130	146	4	5	6	- 96.6	- 95.9	25.0	50.0	2585	2507	2480	- 3.0	- 4.1	-2581	-2502	-2474
50170	262	47	61	57	- 76.7	- 78.2	29.8	21.3	3844	3670	3694	- 4.5	- 3.9	-3797	-3609	-3637
50200	222	70	68	64	- 69.4	- 71.2	- 2.9	- 8.6	3108	3015	3051	- 3.0	- 1.8	-3038	-2947	-2987
50235	229	57	44	43	- 80.8	- 81.2	- 22.8	- 24.6	3421	3389	3399	- 0.9	- 0.6	-3364	-3345	-3356
50270	159	13	24	23	- 84.9	- 85.5	- 84.6	76.9	3828	3730	3741	- 2.6	- 2.3	-3815	-3706	-3718
50290	28	8	23	20	- 17.9	- 28.6	187.5	150.0	2483	2363	2387	- 4.8	- 3.9	-2475	-2340	-2367
50300	3	7	16	15	433.3	400.0	128.6	114.3	1229	1166	1178	- 5.1	- 4.1	-1222	-1150	-1163
50323	10	11	24	25	140.0	150.0	118.2	127.3	2762	2689	2682	- 2.6	- 2.9	-2751	-2665	-2657
50362	14	19	22	25	57.1	78.6	15.8	31.6	2544	2512	2486	- 1.3	- 2.3	-2525	-2490	-2461
50386	16	57	77	77	381.3	381.3	35.1	35.1	206	199	199	- 3.4	- 3.4	- 149	- 122	- 122
50400	80	93	93	96	16.3	20.0	0.0	3.2	67	61	59	- 9.0	- 11.9	26	32	37
50440	507	461	419	440	- 17.4	- 13.2	- 9.1	- 4.6	419	372	345	- 11.2	- 17.7	42	47	95
50500	1129	1070	1022	1053	- 9.5	- 6.7	- 4.5	- 1.6	828	729	690	- 12.0	- 16.7	242	293	363
50534	1012	979	960	953	- 5.1	- 5.8	- 1.9	- 2.7	312	267	270	- 14.4	- 13.5	667	693	683
50560	1117	1091	1067	1070	- 4.5	- 4.2	- 2.2	- 1.9	94	56	60	- 40.4	- 36.7	997	1011	1010
50585	1482	1452	1419	1432	- 4.3	- 3.4	- 2.3	- 1.4	33	9	7	- 72.7	- 78.8	1419	1410	1425
50600	1128	1108	1090	1094	- 3.4	- 3.0	- 1.6	- 1.3	3	4	4	33.3	33.3	1105	1086	1090
50640	3579	3491	3441	3441	- 3.9	- 3.9	- 1.4	- 1.4	0	0	0	0.0	0.0	3491	3441	3441
50700	6061	5407	5392	5348	- 11.0	- 11.8	- 0.3	- 1.1	0	0	0	0.0	0.0	5407	5392	5348
50750	5724	4599	4578	4555	- 20.0	- 20.4	- 0.5	- 1.0	0	0	0	0.0	0.0	4599	4578	4555
50800	6270	4978	4326	4948	- 31.0	- 21.1	- 13.1	- 0.6	0	0	0	0.0	0.0	4978	4326	4948
50900	11976	9995	8642	9954	- 27.8	- 16.9	- 13.5	- 0.4	0	0	0	0.0	0.0	9995	8642	9954
51000	10013	9615	9329	9462	- 6.8	- 5.5	- 3.0	- 1.6	0	0	0	0.0	0.0	9615	9329	9462
51100	9404	8423	8148	8254	- 13.4	- 12.2	- 3.3	- 2.0	0	0	0	0.0	0.0	8423	8148	8254
51170	5433	4921	4741	4784	- 12.7	- 11.9	- 3.7	- 2.8	0	0	0	0.0	0.0	4921	4741	4784
51200	2118	1902	1813	1830	- 14.4	- 13.6	- 4.7	- 3.8	0	0	0	0.0	0.0	1902	1813	1830
51263	4003	3600	3452	3487	- 13.8	- 12.9	- 4.1	- 3.1	0	0	0	0.0	0.0	3600	3452	3487
51289	1419	1315	1268	1278	- 10.6	- 9.9	- 3.6	- 2.8	0	0	0	0.0	0.0	1315	1268	1278
51300	540	515	497	504	- 8.0	- 6.7	- 3.5	- 2.1	0	0	0	0.0	0.0	515	497	504
51342	1837	1766	1695	1726	- 7.7	- 6.0	- 4.0	- 2.3	0	0	0	0.0	0.0	1766	1695	1726
51372	869	805	764	783	- 12.1	- 9.9	- 5.1	- 2.7	0	0	0	0.0	0.0	805	764	783
51400	254	216	206	217	- 18.9	- 14.6	- 4.6	0.5	303	289	278	- 4.6	- 8.3	- 87	- 83	- 61
51415	7	14	6	8	- 14.3	14.3	- 57.1	- 42.9	473	439	422	- 7.2	- 10.8	- 459	- 433	- 414
51428	0	7	0	1	0.0	---	-100.0	- 85.7	561	538	518	- 4.1	- 7.7	- 554	- 538	- 517
51435	0	0	0	0	0.0	0.0	0.0	0.0	310	311	300	0.3	- 3.2	- 310	- 311	- 300
51465	0	1	1	2	---	---	0.0	100.0	1261	1296	1255	2.7	- 0.5	-1260	-1295	-1253
51500	6	2	2	7	- 66.7	16.7	0.0	250.0	1118	1139	1092	1.8	- 2.3	-1116	-1137	-1085
51525	150	147	138	147	- 8.0	- 2.0	- 6.1	0.0	433	432	405	- 0.2	- 6.5	- 286	- 294	- 258
51550	347	332	313	320	- 9.8	- 7.8	- 5.7	- 3.6	189	179	173	- 5.3	- 8.5	143	134	147

Continued Next Page

TABLE I (Continued)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Station	Const. Record Cut	Field Cut	Unadj. Photo. Cut	Adj. Photo. Cut	% Error (4-2)/2	% Error (5-2)/2	% Error (4-3)/3	% Error (5-3)/3	Field Fill	Unadj. Photo. Fill	Adj. Photo Fill	% Error (11-10)/10	% Error (12-10)/10	Net Volume Field	Net Volume Unadj. Data	Net Volume Adj. Data
51580	759	717	682	695	- 10.1	- 8.4	- 4.9	- 3.1	61	37	37	- 39.3	- 39.3	656	645	658
51600	714	688	650	667	- 9.0	- 6.6	- 5.5	- 3.1	2	1	0	- 50.0	-100.0	686	649	667
51635	1209	1164	1094	1122	- 9.5	- 7.2	- 6.0	- 3.6	1	8	4	700.0	300.0	1163	1086	1118
51680	1396	1330	1305	1304	- 6.5	- 6.6	- 1.9	- 2.0	2	10	7	400.0	250.0	1328	1295	1297
51700	580	555	558	547	- 3.8	- 5.7	0.5	- 1.4	0	0	1	0.0	—	555	558	546
51735	727	667	661	663	- 9.1	- 8.8	- 0.9	- 0.6	0	1	0	—	0.0	667	660	663
51760	180	153	152	152	- 15.6	- 15.6	- 0.7	- 0.7	227	210	217	- 7.5	- 4.4	- 74	- 58	- 65
51781	11	16	14	10	- 27.3	- 9.1	- 12.5	- 37.5	610	576	583	- 5.6	- 4.4	- 594	- 562	- 573
51800	0	6	5	7	—	—	- 16.7	16.7	847	823	815	- 2.8	- 3.8	- 841	- 818	- 808
51805	0	1	2	2	—	—	100.0	100.0	247	246	243	- 0.4	- 1.6	- 246	- 244	- 241
51813	0	29	17	17	—	—	- 41.4	- 41.4	408	403	403	- 1.2	- 1.2	- 379	- 386	- 386
51830	0	69	35	33	—	—	- 49.3	- 52.2	938	895	914	- 4.6	- 2.6	- 869	- 860	- 881
51845	0	10	3	2	—	—	- 70.0	- 80.0	844	793	809	- 6.0	- 4.2	- 834	- 790	- 807
51873	0	0	1	1	—	—	—	—	1488	1394	1412	- 6.3	- 5.1	-1488	-1393	-1411
51900	0	4	43	37	—	—	975.0	825.0	1497	1407	1429	- 6.0	- 4.5	-1493	-1364	-1392
51950	0	8	81	69	—	—	912.5	762.5	3320	3130	3161	- 5.7	- 4.8	-3312	-3049	-3092
52000	0	0	1	1	—	—	—	—	3686	3507	3520	- 4.9	- 4.6	-3686	-3506	-3519
52065	0	0	1	0	—	0.0	—	0.0	5019	4793	4779	- 4.5	- 4.8	-5019	-4792	-4779
52100	0	0	0	0	0.0	0.0	0.0	0.0	3593	3114	3204	- 13.3	- 10.8	-3593	-3114	-3204
52127	0	0	0	0	0.0	0.0	0.0	0.0	3010	2660	2733	- 11.6	- 9.2	-3010	-2660	-2733
52163	0	0	0	0	0.0	0.0	0.0	0.0	4014	3859	4006	- 3.9	- 0.2	-4014	-3859	-4006
52200	0	0	0	0	0.0	0.0	0.0	0.0	3550	3343	3514	- 5.8	- 1.0	-3550	-3343	-3514
52250	0	0	2	0	—	0.0	—	0.0	4111	3802	3980	- 7.5	- 3.2	-4111	-3800	-3980
52291	0	0	1	0	—	0.0	—	0.0	2702	2569	2636	- 4.9	- 2.4	-2702	-2568	-2636
52300	0	0	0	0	0.0	0.0	0.0	0.0	579	561	570	- 3.1	- 1.5	- 579	- 561	- 570
52400	3	0	5	1	66.7	- 66.7	—	—	7948	7529	7739	- 5.3	- 2.6	-7948	-7524	-7738
52500	17	0	6	1	- 64.7	- 94.1	—	—	9071	8529	8765	- 6.0	- 3.4	-9071	-8523	-8764
52600	9	0	2	0	- 77.8	-100.0	—	0.0	9865	9437	9642	- 4.3	- 2.3	-9865	-9435	-9642
52700	2	0	4	2	100.0	0.0	—	—	9571	9233	9350	- 3.5	- 2.3	-9571	-9229	-9348
52735	4	1	3	4	- 25.0	0.0	200.0	300.0	4808	4679	4647	- 2.6	- 3.3	-4807	-4676	-4643
52800	4	1	1	2	- 75.0	- 50.0	0.0	100.0	3838	3790	3734	- 1.3	- 2.7	-3837	-3789	-3732
52875	10	0	0	1	-100.0	- 90.0	0.0	—	6337	6386	6320	0.8	- 0.3	-6337	-6386	-6319
52900	6	2	6	9	0.0	50.0	200.0	350.0	1731	1711	1678	- 1.2	- 3.1	-1729	-1705	-1669
52915	3	1	4	6	33.3	100.0	300.0	500.0	909	805	785	- 11.4	- 13.6	- 908	- 801	- 779
52940	1	0	0	0	-100.0	-100.0	0.0	0.0	1835	1666	1662	- 9.2	- 9.4	-1835	-1666	-1662
52980	2	2	0	0	-100.0	-100.0	-100.0	-100.0	2027	1961	1981	- 3.3	- 2.3	-2025	-1961	-1981
53000	61	38	38	38	- 37.7	- 37.7	0.0	0.0	603	585	585	- 3.0	- 3.0	- 565	- 547	- 547
53015	107	80	81	81	- 24.3	- 24.3	1.3	1.3	300	298	298	- 0.7	- 0.7	- 220	- 217	- 217
53045	185	202	187	185	1.1	0.0	- 7.4	- 8.4	443	424	430	- 4.3	- 2.9	- 241	- 237	- 245
53075	157	209	198	196	26.1	- 24.8	- 5.3	- 6.2	237	210	216	- 11.4	- 8.9	- 28	- 12	- 20
53100	242	230	192	200	- 20.7	- 17.4	- 16.5	- 13.0	61	51	50	- 16.4	- 18.0	169	141	150
53120	446	252	207	213	- 53.6	- 52.2	- 17.9	- 15.5	5	5	4	0.0	- 20.0	247	202	209

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TABLE I (Continued)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Station	Const. Record Cut	Field Cut	Unadj. Photo. Cut	Adj. Photo. Cut	% Error $\frac{4-2}{2}$	% Error $\frac{5-2}{2}$	% Error $\frac{4-3}{3}$	% Error $\frac{5-3}{3}$	Field Fill	Unadj. Photo. Fill	Adj. Photo Fill	% Error $\frac{11-10}{10}$	% Error $\frac{12-10}{10}$	Net Volume Field	Net Volume Unadj. Data	Net Volume Adj. Data
53180	1904	1369	1318	1305	- 30.4	- 31.5	- 3.7	- 4.7	0	0	0	0.0	0.0	1369	1318	1305
53200	629	621	617	604	- 1.3	- 4.0	- 0.6	- 2.7	0	0	0	0.0	0.0	621	617	604
53232	1175	1162	1159	1146	- 1.4	- 2.5	- 0.3	- 1.4	0	0	1	0.0	---	1162	1159	1145
53300	3292	3279	3261	3230	- 0.9	- 1.9	- 0.5	- 1.5	0	0	0	0.0	0.0	3279	3261	3230
53350	2721	2734	2716	2660	- 0.2	- 2.2	- 0.7	- 2.7	0	0	0	0.0	0.0	2734	2716	2660
53400	2529	2543	2492	2469	- 1.5	- 2.4	- 2.0	- 2.9	0	0	0	0.0	0.0	2543	2492	2469
53450	2366	2379	2304	2348	- 2.6	- 0.8	- 3.2	- 1.3	0	0	0	0.0	0.0	2379	2304	2348
53500	2406	2402	2338	2371	- 2.8	- 1.5	- 2.7	- 1.3	0	0	0	0.0	0.0	2402	2338	2371
53550	2311	2301	2240	2240	- 3.1	- 3.1	- 2.7	- 2.7	0	0	0	0.0	0.0	2301	2240	2240
53600	2044	2040	1982	1982	- 3.0	- 3.0	- 2.8	- 2.8	0	0	0	0.0	0.0	2040	1982	1982
53650	1561	1540	1484	1484	- 4.9	- 4.9	- 3.6	- 3.6	10	9	9	- 10.0	- 10.0	1530	1475	1475
53700	918	873	817	845	- 11.0	- 8.0	- 6.4	- 3.2	37	28	19	- 24.3	- 48.6	836	789	826
53750	344	308	284	311	- 17.4	- 9.6	- 7.8	1.0	313	291	282	- 7.0	- 9.9	- 5	- 7	29
53800	132	130	143	148	8.3	12.1	10.0	13.8	830	788	771	- 5.1	- 7.1	- 700	- 645	- 623
53820	57	62	67	66	17.5	15.8	8.1	6.5	373	345	349	- 7.2	- 6.4	- 311	- 279	- 283
	161,856	147,564	143,504	143,757	- 11.3	- 9.9	- 3.5	- 2.0	252,303	246,002	247,360	- 2.5	- 2.0	-103,639	-102,478	-103,588
0+00	BP-3															
0+50	141	181	214	204	51.8	44.7	18.2	12.7								
1+00	617	632	644	657	4.4	5.7	1.9	3.2								
1+50	1638	1693	1658	1758	3.1	7.3	- 2.1	3.8								
2+00	3197	3283	3182	3309	- 0.3	3.5	- 3.1	0.8								
2+50	4890	4749	4617	4714	- 1.4	0.5	- 2.8	- 0.7								
2+76	2977	3047	3004	3052	0.9	2.5	1.4	0.2								
	13290	13585	13319	13694	4.4	3.2	- 2.0	0.8								
530+50	BP-4															
531+00	3577	3680	2882	2790	- 19.3	- 21.9	- 21.7	- 24.1								
531+20	1600	1664	1503	1519	- 6.2	- 5.1	- 9.9	- 8.8								
531+43	2113	2398	2210	2247	4.6	6.3	- 7.6	- 6.2								
532+00	5548	5630	5342	5368	- 3.7	- 3.2	- 5.1	- 4.7								
533+00	6932	8239	7970	7991	15.0	15.3	- 3.3	- 3.0								
534+00	2806	4265	4050	3935	44.3	40.2	- 0.4	- 3.3								
534+40	1112	1141	1107	1121	- 0.4	0.8	- 3.0	- 1.8								
535+00	1824	1872	1766	1872	- 3.2	2.6	- 5.7	0.0								
535+50	993	1015	956	969	- 3.7	- 2.4	- 5.8	- 4.5								
536+00	356	377	326	328	- 8.4	- 12.9	- 13.5	- 17.8								
	26861	30079	28109	28124	4.6	4.7	- 6.5	- 6.5								

TABLE II

Homogeneity of Variance Among Eleven Models
(Bartlett's Test)

HYP.: $\sigma_i^2 = \sigma_2^2 = \sigma_{BP-4}^2$, ALT. HYP.: Not All Equal, ($\alpha = .05$)

Model	s_i^2	ν_i^2	LOG s_i^2	$\nu_i s_i^2$	ν_i LOG s_i^2
1	.0984	19	8.99300-10	1.870	170.86700-190
2	.0144	25	8.15836-10	.360	203.95900-250
3	.0629	24	8.79865-10	1.510	211.16760-240
4	.0377	22	8.57634-10	.829	188.67948-220
5	.0267	15	8.42651-10	.401	126.39765-150
6	.0250	31	8.39794-10	.775	260.33614-310
7	.0331	35	8.51983-10	1.159	298.19405-350
8	.0974	19	8.98856-10	1.851	170.78264-190
9	.0213	22	8.32838-10	.469	183.22436-220
BP-3	.0633	6	8.80140-10	.380	52.80840-60
BP-4	.0750	10	8.87506-10	.750	88.75060-100
		228		10.354	1955.16692-2280

$$s^2 = \frac{\sum \nu_i s_i^2}{\nu} = \frac{10.354}{228} = .0454 \qquad \text{LOG } s^2 = 8.65706\text{-}10$$

$$\beta = \frac{2.303}{C} [\nu \text{ LOG } s^2 - \sum \nu_i \text{ LOG } s_i^2]$$

$$\beta = \frac{2.303}{C} [228(8.65706\text{-}10) - (1955.16692\text{-}2280)] = \frac{42.93}{C}$$

$$C = 1 + \frac{(\frac{1}{\nu_i}) - \frac{1}{\nu}}{3(K-1)} = 1 + \frac{.6722 - .0044}{3(10)} = 1.022$$

$$\beta = \frac{42.93}{1.022} = 42.01, \quad \chi^2_{10,.05} = 18.31$$

$42.01 > \chi^2_{10,.05}$

CONCLUSION: The variances are significantly different.

TABLE III

Homogeneity of Variance Within a Model

Elevation Data

Reading	Point 1	2	3	4	5	6	7	8	9
1	27.0	32.3	10.5	60.9	47.9	20.5	87.3	83.4	73.8
2	26.6	32.4	10.3	61.0	48.0	20.8	87.2	83.4	73.8
3	26.8	32.6	10.5	61.0	48.0	20.6	87.4	83.4	73.9
4	27.1	32.2	10.3	60.8	47.9	20.8	87.3	83.3	74.0
5	26.5	32.4	10.3	60.7	47.9	20.7	87.1	83.1	73.7
6	26.5	32.2	10.4	60.8	47.9	20.6	87.2	83.3	73.7
7	26.6	32.3	10.0	60.7	47.8	20.5	87.1	83.1	73.8
8	27.1	31.9	10.5	60.8	47.7	20.5	87.2	83.1	73.8
9	26.6	32.4	10.3	60.8	47.7	20.5	87.3	83.1	73.7
10	26.5	32.0	10.1	60.6	47.8	20.5	87.1	83.2	73.8

Bartlett's Test

HYP.: $\sigma_1^2=\sigma_2^2=\ldots\sigma_9^2$, ALT. HYP.: Not All Equal, (α = .05)

Point	s_i^2	LOG s_i^2
1	.0622	8.79379-10
2	.0422	8.62531-10
3	.0231	8.36361-10
4	.0167	8.22272-10
5	.0111	8.04532-10
6	.0156	8.19312-10
7	.0111	8.04532-10
8	.0177	8.24797-10
9	.0089	7.94939-10
Total	.2086	74.48655-90

TABLE III (Continued)

$$s^2 = \frac{\sum \nu_i s_i^2}{\nu} = \frac{9(.2086)}{81} = .0232$$

$$\text{LOG } s^2 = 8.36549\text{-}10$$

$$\beta = \frac{2.303}{C}\,[\nu \text{ LOG } s^2 - \sum \nu_i \text{ LOG } s_i^2]$$

$$\beta = \frac{16.635}{C}$$

$$C = 1 + \frac{\sum (\frac{1}{\nu_i}) - \frac{1}{\nu}}{3(K-1)} = 1 + \frac{1 - .0123}{3(8)} = 1.0411$$

$$\beta = \frac{16.635}{1.0411} = 15.98$$

$$\chi^2_{8,.05} = 15.51, \qquad 15.98 > \chi^2_{8,.05}$$

CONCLUSION: Variances are significantly different at the five percent level.